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THE MECHANICAL POWER OUTPUT OF MEN

by

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FOREWORD

This report has been prepared for the Office of Naval Research under Contract No. Nonr-2180(00) entitled, "Research on Human Power Output".

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The author would like to thank Dr. A. E. Hickey, Jr. for making previously unpublished data available for this report and Dr. A. W. Raspet for his help in obtaining data from a report of the Muskelflug Institut.

Finally, apologies are offered to the many authors of the prime sources referenced in this report for the treatment to which their data were subjected in the following pages.

ABSTRACT

A scheme for designing man-powered devices for optimal power transfer from the human operator to the mechanism is discussed. Data indicating the feasibility of such a design are presented.

Such data as were available for unusually high as well as for average power production in cranking, pedalling, and other tasks are presented in a systematic fashion.

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SECTION I

INTRODUCTION

Man is a gregarious tool-using animal. Within this definition of our species lies the central problem of engineering psychology: how do men function in company with other men, in company with machines, or with both men and machines? In this paper we will be concerned with a restricted phase of man's tool-using activity; namely, his ability to perform work in the physical sense of the word.

This restriction to the performance of physical work by man means that we will be dealing in energetics rather than dynamics. To study the dynamics of the power-producing man would require a detailed description of all the forces generated, their points of application, and the structural constraints. Although work has been begun along these lines, such a detailed analysis is beyond the scope of this report. A discussuion of energies is considerably easier, since one can deal with average, scalar quantities. In order to achieve this simplicity of description, we pay a price. This price is the loss of the detailed time history of the process which a dynamic study would afford. Actually, we will discuss dynamics in Section II of this report. We will do this, in order to gain insight into methods by which energy transmission may be improved. The measured data which will be presented subsequently, however, will be almost exclusively measurements of power and work.

By work is meant the product of a force and the distance along which it is directed. The units in which work is expressed are those of energy, and thus work can be measured by increases in mechanical potential energy, kinetic energy, chemical or electrical energy, heat, and even light and sound. Work may be useful or it may be dissipated and wasted. By useful work we mean work which achieves a change external to the energy-producing device which is desirable according to some given criterion. Internal work, like internal power losses in a battery, is undesirable.

The quantity with which this report will be mainly concerned is power. Power is defined as the time rate of doing work.

There are many different units for expressing power. The most common are watts, kilocalories/minute, foot-pounds/minute, kilogram-meters/minute, and horsepower. In the following pages, we shall use horsepower as the standard unit almost exclusively, since this unit has a certain intuitive appeal. As used in this report, one horsepower is defined as 550 foot-pounds/second. There are other definitions of horsepower which are approximately, but not exactly, equal to the foregoing.

The common definition in this country and Great Britain of one horsepower being equal to 746 watts is equivalent in practical matters to our working definition, since 550 foot-pounds/second equals 745.7 watts. The horsepower used in continental Europe is called metric horsepower. Since 1.014 horsepower = metric horsepower, this definition is sufficiently close to 550 foot-pounds/second for any of the purposes of this report.

When the tools or machines with which man worked were rather primitive the major purpose in using a man to operate a machine was to exploit his ability to generate mechanical power in a controlled fashion. As technology advanced, it became less important that the human operator be capable of generating power, and more important that he function as a control element. Consider Figure 1-1 which is a diagrammatic presentation of man as a controller and power source.

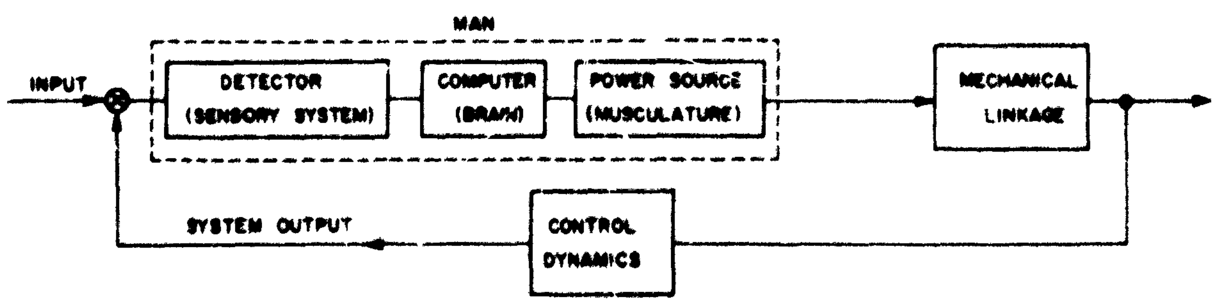


Figure 1-1b. Closed Loop Control

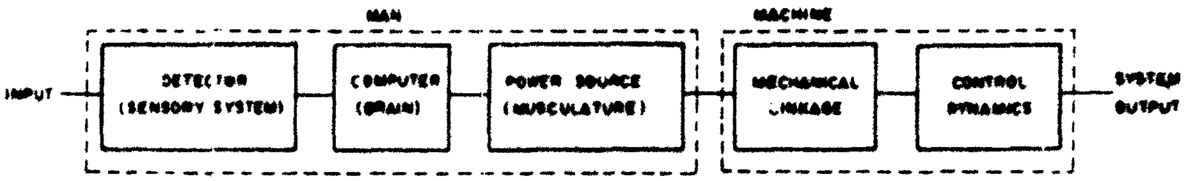


Figure 1-1a. Open Loop Control

In the intact organism it is impossible to completely isolate the human operator's control functioning from his power-generating functioning since any measurements will involve some form of power-produced output. We can, however, as a practical matter, distinguish between those functions of the human operator which are primarily control and those functions which are primarily power production. This distinction parallels the commonly used division in electrical engineering applications between communications engineering and power engineering. Thus, we are examining complementary aspects of man; the energy-producing controller.

There exists at present a highly organised body of knowledge about the control functions of man. By and large, the data basic to this body of knowledge were gathered under conditions such that the control dynamics in Figure 1a or 1b contained a source of power considerably in excess of the power inherent in a man's musculature. Thus, whether performing an open loop task like lowering an aircraft's wheels before landing, or whether performing a closed loop task like training a naval gun, the human was operating so that his strength was amplified by the mechanism under control. The interest in this type of human activity was a logical consequence of the extended evolutionary development wherein man learned to harness and to create power sources. As power sources

became more complicated and the operator's relationship with these sources became more remote, the problem of controlling these power sources with relatively minor outputs of energy increased in importance.

When technology was rather primitive, the amount of power a man could generate and the duration for which he could maintain it were of interest. Many manpower-operated devices devised in ancient times evolved to what is probably a reasonably efficient level of performance. Consider the technique used by the ancient Egyptian, and still in use today, to draw water from the Nile for irrigation by means of a device called the shadoof.

The shadoof is a man-powered counterpoised sweep used to lift water and deliver it into a channel. The water is carried in a bucket which is lashed to the base of an upright pole. The upper end of this pole is attached by means of a rope to one end of a light, fairly rigid beam. At about three quarters of the distance to the lower end of this beam a wooden pivot pin passes through a hole in the beam. This pin is suspended by short cords of rope attached to a sturdy wooden bar; the ends of which are supported by two upright pillars of wood or reinforced mud. A globular mass of dried mud and chopped straw is plastered around the other end of the beam to serve as a counterpoise. This counterpoise of about 230 pounds is heavy enough to raise a bucketful of water suspended from the other end of the beam.

The system operates by the man pulling down on the light upright pole, imparting potential energy to the counterpoise, and releasing the pole so that the counterpoise in releasing its energy raises the bucket. The man does work by bending his body to extreme flexion when pulling down on the bucket and straightening to the upright position as the bucket ascends. The customary work scheduling used is for two men to work alternately at the rate of about $6\frac{1}{2}$ buckets per minute; each working 6 hours daily. If we consider the bucket to weigh 60 pounds, and it is lifted 11 feet, each man produces 0.13 hp per 6-hour day. On the

other hand, if we consider the work the man does on the counterpoise, which is his actual external work output, it is about 0.15 hp per 6-hour day.

Since the shadoof has been used for at least 3000 years, it is reasonable to believe that it represents a design which is optimal in some sense. Very possibly, were we to consider the overall energy relations in terms of caloric intake, metabolism, and useful work output, the particular work scheduling and mechanism which was evolved represents a peak of efficiency achieved by evolution. It is entertaining to wonder whether the oscillating man-shadoof system approaches a resonant condition with the energy interchange occurring between the active human element and the passive globular blob.

Another example of man power generation under the implied restriction that fatigue be kept within reasonable bounds, an unfortunately hazy notion, resulted from Coulomb's thoughts on the subject(15).

Coulomb held that the greatest work which a man can perform each day without undue fatigue consists in raising his own body. The simplest contrivance meeting Coulomb's requirements was used by Capt. Coignet in the construction of the earthworks of a fort near Paris. The device consisted of a pulley having boards at either end. One board carried the man and the other the load which weighed slightly less than the man. The man clambered up the fortification wall and stood upon the board. His weight then caused the weight to rise as he descended. The average power generated by this method was 0.13 hp over an 8-hour day. Each worker raised his own body weight (about 70 kgms) 13 meters high, 310 times per day.

Poncelet and General Morin compiled a table for manual work from which the following entries were drawn and which includes Coignet's data(50).

Table 1-1 MANUAL WORK FROM PONCELET AND MORIN

	Man's Activity	Duration	Average Horsepower
1.	Raising his own weight up stair or ladder	8 hours	0.13
2.	Hauling up weight with rope	6 hours	0.055
3.	Lifting weights by hand	6 hours	0.044
4.	Carrying weights upstairs	6 hours	0.034
5.	Shoveling up earth to a height of 5 feet, 3 inches	10 hours	0.014
6.	Wheeling earth in barrow up slope of 1 in/ft, horiz. veloc. 0.9 ft/sec (return empty)	10 hours	0.018
7.	Pushing or pulling horizontally (capstan or oar)	8 hours	0.096
8.	Turning a crank or winch	8 hours	0.082
9.	Turning a crank or winch	2 minutes	0.524

Additional data on human power output on a daily basis were gathered in England as the industrial revolution gathered momentum and economic competition became keener.

In these and many other assessments of the capabilities of men to do work, the output of the mechanism in terms of a weight lifted a certain number of feet in a given length of time provided the measure of the man's power. Since a direct measure of the man's work was not made, that part of the man's energy which was required to overcome frictional resistance in the mechanism cannot be specified accurately. As a result, we have no good idea of what the man's output really was. Clark, however, suggests, that as a rule of thumb, one-third of the man's output was dissipated in overcoming resistive losses. We have denoted this compensated output as actual generated horsepower in the following paraphrased data (14).

> A Mr. Smeaton concluded that "good English labourers" could produce the equivalent of 3904 foot-pounds of work per minute; and this was estimated to be about twice the output of ordinary persons "promiscously picked up." The task on which Mr. Smeaton made his test was one of pumping water and the system power output was 0.118 horsepower or actual generated horsepower of 0.157. The time interval was presumably 8 hours.

Mr. John Walker found that an ordinary laborer working daily at a winch or crane handle could exert an average force of 14 pounds moving at the rate of 220 feet per minute equivalent to a system output of 0.093 horsepower or average, actual generated horsepower of 0.124.

Under conditions similar to Mr. Walker's, a Mr. Glynn asserted that a man-machine system could generate 0.167 horsepower for short periods of time and 0.10 horsepower continuously. His data differed from Mr. Walker's in that he claimed that the subjects could exert 25-pound forces and 15-pound forces on the crane handle under bursts of effort and steady-state conditions. The corresponding actual generated horsepowers are 0.223 and 0.13.

A Mr. G. B. Bruce found that a laborer in average work at a pile driver exerts a force of 16 pounds, plus overcoming the resistance of the gearing, at a velocity of 270 feet per minute, for 10 hours a day, making one blow every four minutes. The average power is thus 0.131 horsepower or actual generated horsepower is 0.175

In 1826, Mr. Joshua Field, as reported by Clark, tested the performance of men at a crane of rough construction. The handle was eighteen inches long and the loads on the handle varied from 10 to 35 pounds. His results are presented in Table 1-2.

Mr. Field stated that experiment No. 4 gave a near approximation to the maximum for human power generated over 2.5 minutes. He found that in the succeeding trials the men were so exhausted as to be unable to let down the load.

Table 1-2 MANUAL PERFORMANCE AT A CRANE

Exp. No.	Static resistance of load on the handle in pounds	Time in raising load (min)	Useful horsepower output	Actual generated horsepower	Remarks
1	10	1.5	0.350	0.467	Easily done by a stout Englishman.
2	15	2.25	0.349	0.465	Tolerably easily by the same man.
3	20	2.0	0.525	0.700	Not easily by a sturdy Irishman.
4	25	2.5	0.525	0.700	With difficulty by a stout Englishman.
5	30	2.5	0.630	0.840	With difficulty by a London man.
6	35	2.2	0.833	1.111	With the utmost difficulty by a tall Irishman.
7	35	2.5	0.736	0.981	With the utmost difficulty by a London man.
8	35	2.83	0.650	0.867	With extreme labour by a tall Irishman.
9	35	3.0	0.612	0.816	With very great exertion by a sturdy Irishman.
10	35	4.05	0.458	0.611	With the utmost exertion by a Welshman.
11	35	----	-----	-----	Given up at this time by an Irishman.

Although these early data leave much desired, it is clear that many of the major problems in human power output were recognized by these early observers. Thus, both Bruce and Glynn and the ancient Egyptians were cognizant of time-scheduling problems, and Field certainly recognized population sampling problems.

Our present interest in man as a power generating device has two bases. First, we have a desire to be able to integrate our knowledge of how man's energy resources function with our knowledge of how man operates as a control device. In so doing, we will come closer to achieving a description of man. Without a dynamic discussion of the actuators, however, our description will be lacking. Second, there are practical problems which

arise when man is called on to generate power. Such demands can occur under the following conditions:

(1) For either safety or military reasons it may be necessary for man to be capable of operating certain devices when other power sources fail.

(2) Limitations of space and weight as would occur in space vehicles may be such that power augmentation is logistically expensive. If the human is a passenger acting as an observer he may have to act as sensor, controller, and to a modest extent, power source as well, to pay his way.

SECTION II

POWER TRANSFER

In order to gain some perspective on a mechanical system utilizing human power, it will be worthwhile to make some general statements about the transfer of power to a useful load (45). In Figure 2-1, we have a human power source connected in some unspecified manner to a load; i.e., some device on which work is to be done. The force output of the man is $f_M(t)$.

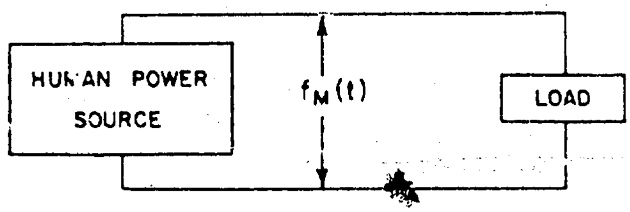


Figure 2-1. Block Diagram of Man Transferring Power to a Load

We can characterize the human power source as having an internal impedance, Z_M, composed of resistance, R_M, and a reactance, X_M. Similarly, we can characterize the load as having an impedance, $Z_L = R_L + X_L$ and $R_L = R_{UL} + R_{FR}$, where R_{FR} is the resistance in the load due to frictional efforts and R_{UL} is the resistance characterizing the useful load. We will leave in abeyance the question as to whether once characterizing a man in this manner we can ever obtain relevant measurements. Let us consider two simple cases:

First, let $f_M(t)$, the man's force output, be equal to a constant F_M, and let us consider $X_M = X_L = 0$. The significance of allowing the reactances to be zero is that this assumed system does not contain energy storage devices. Under these conditions the power transmitted to the load, P_L, can be expressed as follows:

$$P_L = \frac{F_M^2 R_L}{(R_M + R_L)^2} = P_{UL} + P_{FR} = \frac{F_M^2 R_{UL}}{(R_M + R_{FR} + R_{UL})^2} + \frac{F_M^2 R_{FR}}{(R_M + R_{FR} + R_{UL})^2} \quad (2\text{-}1)$$

(a) Since the second term in the above expression represents wasted power, we clearly want to design our system so as to minimize R_{FR} under all conditions.

(b) Consider R_M, the man's internal resistance, as a variable under our control. Such a condition may arise as a consequence of a work scheduling such that fatigue products are minimized, or it may arise by properly choosing the man's location with regard to his machine, or by a proper choice of human output. Equation (2-1) states that the useful power delivered to the load will be at a maximum for R_M as small as possible.

(c) Consider the useful resistive load itself, R_L, as being a variable under our control. This is, of course, the most common circumstance which we might encounter in practice. Our useful power is then

$$P_{UL} = \frac{F_M^2 R_{UL}}{(R_M + R_{UL} + R_{FR})^2}, \tag{2-2}$$

and this expression is maximized, subject to the above conditions for

$$R_{UL} = R_M + R_{FR}. \tag{2-3}$$

The requirement to minimize R_{FR} still obtains, of course.

The foregoing illustrates that it is reasonable to postulate a maximum power transfer condition for a human operator generating a steady force output. However, when attempting to apply the concept of matching impedances illustrated in Equation (2-3), one is confronted with the problem that neither F_M nor R_M are independent of one another; nor are they independent of time. The circumstances under which a man exerts a force determines the time history of this force and the time history of his internal resistance and their mutual interdependence.

One can elaborate Equation (2-3) slightly by assuming that the force which the man produces, f_M, is a function of time. For simplicity, let us consider a sinusoidal time dependency such that

$$f_M = F_M e^{j\omega t} \tag{2-4}$$

This assumption of a harmonic force output of frequency is not very restrictive, since a harmonic output is a reasonable approximation to an on-off duty cycle type of operation. The period of the duty cycle can be approximated by the fundamental of the harmonic analysis used, and the higher harmonics can often either be neglected or lumped together and treated as a residual error in the mathematical model chosen.

The harmonic output described in Equation (2-4) could be achieved without periodic work scheduling if the human operator in Figure 1-1 is defined so as to include the physical device through which he applies force. If the power-generating system were defined as a man and a wheel which he was rotating, then the conversion of this system's output to a linear motion would produce a harmonic force output. In time, of course, we would expect an attenuation of this output and we might expect a force whose time history could be approximated by

$$f_M = F_M e^{(j\omega - \alpha)t} \tag{2-5}$$

The decay term, , would be a function of the conditions under which power was exerted as well as of the physiology, and perhaps psychology, of the man generating the power.

Essentially the same reciprocating effect can be obtained by operating such mechanisms as a pump handle. In such an example there is not a true zero output on the man's part during the "off" cycle, but the "on" cycle power is much greater than the "off" cycle power.

The useful purpose served by introducing a model for force output which has a periodic time characteristic is that we can elaborate the previously used model for the man's impedance, Z_M, and the load's impedance, Z_L, by including the reactance terms. Reactance elements can be thought of on an intuitive level as devices which can store and then release energy. Thus, one can see that if energy is supplied to a system in a periodic fashion it might be possible to arrange reactance elements in such a manner that a smooth energy output would result. Furthermore, one would expect that the optimal power transfer conditions stated for pure resistive loads would be altered.

Equation (2-1) becomes the following when we include the effects on the system of energy storage devices:

$$P_L = P_{UL} + P_{FR} = \frac{F_M^2 R_{UL}}{\left[(R_M+R_{UL}+R_{FR})^2 + (X_M+X_L)^2\right]^{\frac{1}{2}}} + \frac{F_M^2 R_{FR}}{\left[(R_M+R_{UL}+R_{FR})^2 + (X_M+X_L)^2\right]^{\frac{1}{2}}}$$

(a) As before, it is clear that R_{FR}, the frictional resistance, should be as small as possible.

(b) If we consider R_M as the variable with respect to which we maximize P_{UL}, one can show that this maximisation occurs when

R_M is as small as possible.

If, in addition, we can control X_M and X_L then the maximum occurs for

$$X_M = - X_L$$

(c) If we consider the useful resistive load, R_{UL}, as being a variable under our control, then maximum power transfer conditions become:

$$R_{UL}\left[(R_M+R_{FR})^2 + (X_M+X_L)^2\right]^{\frac{1}{2}} \tag{2-7}$$

where, as before, R_{FR} is to be made as small as possible.

(d) If one or both reactances are adjustable then $X_M = -X_L$ is another maximising condition.

As with the constant force output condition, we have the problem of parameters in the power transfer expression which are dependent both on the conditions of exertion and on time. The human's mechanical reactance, X_M, however, is somewhat less dependent on the foregoing factors than is the resistive term R_M. This follows from the definition of X_M in terms of system components:

for rotational motion

$$X_{M_\phi} = j(\omega I_M - \frac{1}{WC_{M_\phi}})$$

for rectilinear motion (2-8)

$$X_{M_\rho} = j(\omega M_M - \frac{1}{WC_{M_\rho}})$$

I_M is the moment of inertia, and M_M is the mass for that portion of the operator's anatomy which is involved in generating power. Clearly this parameter, though dependent on the technique selected for generating power, is relatively insensitive to time effects. The compliance terms, C_{M_ϕ} for rotational motion, and C_{M_ρ} for rectilinear motion may change with both the operator's position and with time. This is because the resiliance or springiness of the anatomy is a function of muscle tone which is in turn

presumably governed by such time dependent effects as fatigue.

The foregoing development has provided us with a rudimentary theory for the synthesis of human power generating devices for maximal power transmission to a useful load. Nothing has been stated, however, about the efficiency of the human operator himself in terms of his energy input and output, and how this efficiency is related to his method of generating power. In addition, we have not considered the possibility of an interaction between generating power and performing a control task simultaneously. The foregoing restrictions imply that the optimal man-power arrangement will be a rather complicated compromise, depending on the specific needs of the problem at hand.

We can, however, proceed to outline the types of information which we will need about the human operator if we are to attempt the design, from the foregoing basis, of a man operated, power-generating device.

Primarily, we must have measures of the human operator's forces, compliances, resistances, masses, and inertias as they exist in various postural geometries and as they vary with time.

First, we will need complete information on how the force exerted by a human operator is dependent on the time schedule of his efforts. Starting with a continuous effort it would be useful to know the duration that this effort can be sustained at a constant level and the form of the decrement of this effort with time. The confounding of such a finding with the means by which the human exerts power present a difficulty common to all the measurements of the scheduling of human efforts. The means for exerting forces subsumes both the mechanical device through which the human is functioning as well as the posture or stance and, particularly, muscular output which characterize the human. It would be a pleasing simplification if the time characteristics describing the decrement with time of the human operator's output were essentially similar in shape without regard to the means of exerting forces. The actual means of exerting power might then establish a scale factor from which a family of curves would arise.

Time scheduling can be elaborated to the duty cycle type of operation previously discussed. Figure 2-2 may make the variables associated with duty cycle operation more obvious.

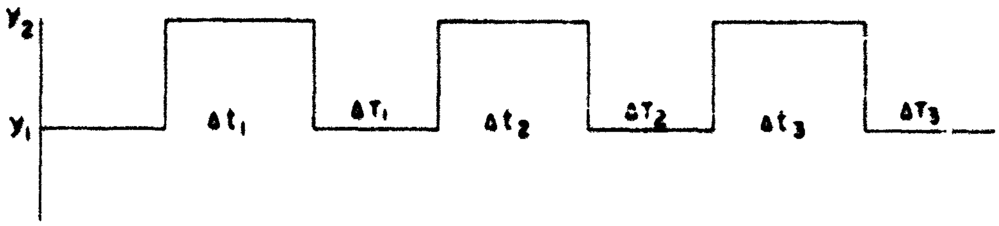


Figure 2-2. Time Schedule of Manual Efforts

(a) The term $y_2(t) - y_1(t)$ is the power generated in the "on" cycle. The base level, y_1, is not necessarily zero.

(b) The term Δt_1 denotes the "on" time of the first cycle, and Δt_i of the i^{th} cycle.

(c) The term ΔT_1 denotes the "off" time of the first period immediately following the first "on" cycle, and ΔT_i of the i^{th} period.

The boxcar shape, $y(t)$, shown in Figure 2-2, is one of many possible functions. The "on" time functions could be arcs of a circle; sinusoidal; periodic or random; etc.

Some of the questions relating to force output under the conditions shown in Figure 2-2 are:

(a) What magnitudes should y_2 or y_1 assume?

(b) Is there an optimum $\Delta t, \Delta T$ relationship from either the viewpoint of metabolic efficiency or average power output?

(c) How do y_2 and y_1 affect the Δt, ΔT relationship?

(d) What form should $y(t)$ assume?

A third type of human force output related to time scheduling is impulsive force. Impulsive forces are generated when the human operator releases a large burst of energy over a short time interval in a task which does not require a continuous or serial type of energy outlay. Some examples of this type are: kicking a football, batting a ball, pushing, pulling, or twisting under appropriate conditions. In general, impulses are best handled by the momentum equation,

$$I = \int_0^T F_{dt}.$$

The mechanical characteristics of the human operator in various stances and postures will depend in some not readily predictable manner on the time scheduling. For example, resistance or compliance might be altered by the onset of fatigue, which in turn could be related to time scheduling.

A third major area where information is needed relates to a constellation of nebulous situational and individual difference effects. Clearly,the implied development of useful data for engineers concerning human power generation capacity must face the problem of individual differences. Similarly, motivation, social facilitation, general physical environment, and so forth, will all have an effect on the human's power output.

Since the goal of this report is to present a collection of existing data on power generation and to present these data in a logical order, the foregoing considerations on the dynamics of the human force-generating device will be used to organize available data on human power output. The paucity of the data will make the foregoing analysis appear overrefined. Nevertheless, such an analysis should be latent in evaluating and interpreting the available data.

SECTION III

CHARACTERISTICS OF STRIATED MUSCLE

Although human power generation could be studied on a gross level without any detailed notions of internal functioning, the description, in Section II, of human dynamics in terms of components capable of storing or releasing energy implies that a detailed notion of the functioning of the man's muscles might shed some light on the validity or range of applicability of this description.

The thesis that gross muscular behavior could be predicted in terms of the behavior of isolated striated muscles, was the guiding principle for a series of inspired experiments conducted over the last four decades by A. V. Hill and his colleagues(4;20;21;28;35-39;44;45;59-61). Originally, the central concept in this work was that striated muscle tissue, without regard to its biological origin, had a "visco-elastic" property and could be described as a mechanical system. This early lack of emphasis on the physical chemistry of muscles proved to be the undoing of this theory when it was used to estimate the efficiency of muscular activity. The extent of the postulated viscosity was thought to be related to the origin of the muscle and to its state of activity. This property of muscle tissue was presumed to be demonstrated by so-called quick-release experiments whereby an excised muscle, or intact muscle grouping subjected to isometric tetanus, was suddenly caused to shorten slightly. The characteristic muscular response was for the tension to drop and then slowly rise to its final equilibrium value along a path identical in shape with the path characterizing its initial excitation. This behavior is typical of the transient response of a visco-elastic system. Thus, Hill was able to express the external force, F, exerted by the excised muscle in terms of the theoretical maximal force, F_o, the contraction time, t, and the coefficient of viscosity, k, as:

$$F = F_o(1 - k/t) \tag{3-1}$$

In Hill's usage, $1/t \propto V$, where V is the velocity of shortening of the muscle; hence k may be associated with a viscous resistance in Equation (3-1). In applying Equation (3-1), or subsequent elaborations, it should be noted that this equation applies for maximal effort contractions. Remarkably good fits of the experimental data were obtained for Equation (3-1) for a variety of excised animal muscles as well as for the gross behavior of several complex muscle groupings in human subjects. Perhaps the major point to be noted is that since we can reasonably expect a decay of human mechanical energy production with time, many kinds of muscular performances may be expected to fit a curve of the form, Equation (3-1), in some average sense. Whether such gross measures actually indicate individual muscle action will become increasingly more doubtful as the task becomes more complicated, since innervation and muscle groupings effects will become of importance.

Fenn(25,26) questioned Equation (3-1) and maintained that the change in true internal muscular force, F_t, as a function of the velocity of contraction was proportional to the tension in the muscle so that

$$-\frac{dF_t}{dv} = aF_t,$$

where

$$F_t = F_o e^{-aV} = F + cV$$

therefore,

$$F = F_o e^{-aV} - cV, \qquad (3-2)$$

where F is the external force exerted, or the load on the muscle, and F_o is the theoretical maximum force which is exerted at zero velocity. The constant, a, is a coefficient of tension loss which is fairly uniform over different muscles. The constant, c, is the coefficient of viscosity.

Fenn and Marsh studied the foregoing relationships for the satorious and gastrocnemius muscles of Rana pipiens (leopard frog), as well as for the gastrocnemius muscles of cats, and found it to be substantiated by their curve-fitted data. In fact, the relationship holds for muscles in dog-fish, scallops, and Phascolosoma worms as well.

It is important to note that Fenn's data showed Equation (3-2) differed from Equation (3-1) only at higher velocities of shortening than those for which Equation (3-1) was originally studied. Additionally, excised muscles freed from any nervous influence, were the exclusive test material for Equation (3-2). Equation (3-1) was verified for muscles in situ in the performance of gross tasks, e.g. human arm muscle, and yielded data which is more pertinent for our purposes. It is possible that the basic difference between Fenn's Equation (3-2) and Hill's Equation (3-1) resulted from:

(a) The reflex innervation of the controlling muscles with the corresponding complicated phasing of the various discharges.

(b) The fact that in Hill's work on intact organisms the measurement of V was determined for a limb, or for the entire organism, not for the individual muscle fibres in question.

It is of interest to examine the disparities between Equations (3-1) and (3-2) by expanding e^{-aV} in a power series and neglecting quadratic terms and higher. In this case, letting $c = F_o c'$, Equation (3-2) becomes:

$$F = F_o \left[1 - (a+c') V + \ldots\right]. \tag{3-3}$$

Letting $V = K/t$, Equation (3-1) becomes:

$$F = F_o \left[1 - k\,V/K\right]. \tag{3-4}$$

Since Equation (3-3) is formally identical with Equation (3-4), Fenn's correction to Equation (3-1) is a second order effect. Assuming a proper scaling, the quadratic term in Equation (3-4) would become of importance as $(a + c')V$ became greater than unity. This fits the experimental facts quite well.

Fenn's major point, however, was that the muscle cannot be considered naively as a purely mechanical system. He maintained that the loss of tension with increasing shortening speed is a chemical rather than a frictional effect. For example, a muscle releases energy while maintaining a contraction. This is highly aberrant behavior for the usual passive mechanical spring!

In later work(34,36), which was influenced by Fenn's contention, Hill became convinced that the visco-elastic model accounted for, at best, a minor part of the description of muscular behavior. He then introduced concepts which recognized the physico-chemical as well as the mechanical nature of muscles.

A finding from these later experiments was that an active muscle appears to be a two-component system consisting of an undamped, purely elastic element in series with a contractile element. The following empirical relationship was found by Hill to characterize isotonic shortening:

$$(F + a)V = b(F_o - F), \qquad (3\text{-}5)$$

or alternatively,

$$(F + a)(V + b) = (F_o + a)b = \text{constant}.$$

where (1) a is a constant expressing the heat liberated in contraction above that liberated by an isometrically-mounted, electrically-tetanized muscle; i.e., "ax" equals the heat of shortening.

(2) b defines the absolute range of energy liberation, and "dx/dt" is the muscle velocity of contraction.

(3) F_o is full isometric tension, and "F" is the load or external force exerted.

(4) V is the velocity of shortening of the muscle fiber.

Equation (3-5) represents the present best approximation to muscle behavior as has been demonstrated by a large number of experiments for different types of animal tissue in excised as well as intact organisms. It should be observed that Equations (3-1), (3-2), and (3-5) all have approximately the same shape over limited ranges.

Without the need to accept the early concepts for the force-velocity relations characterising individual muscles, we can make use of certain of the carefully measured data on gross muscular behavior which was collected by Hill's school.

The capacity of the elbow muscles to do work in a maximal effort contraction was measured experimentally by Hill and Lupton(44,45). The technique consisted of opposing the force exerted by the flexion of the elbow in maximal isometric contraction with the reaction of a flywheel of variable moment of inertia. The kinetic energy imparted to the wheel of known size and mass was measured by the quick-release technique. It was observed from experimental data that for both Hill's earlier study, and the more thorough succeeding work by Lupton:

$$W = W_o(1 - k/t), \qquad (3\text{-}6)$$

where W was measured in foot pounds, and t, in seconds. Equation (3-6) follows from Equation (3-1) on the assumption that the force is constant over the distance of shortening; i.e., W = Fx.

Hill identified:

(1) W as the external work performed in maximal effort contraction.

(2) W_o as the work done in a similar isometric contraction as the velocity of contraction approached zero.

(3) k as the shortest possible time in which the muscle could contract.

The average of nine measurements on each condition for a practiced subject yielded the numerical values for Equation (3-6) shown in Equation (3-7). Work is in foot pounds, and time, in seconds. The variability between subjects altered the constants only slightly:

$$W = 79.1\,[1 - 0.26/t] \qquad 0.26 \leq t \qquad (3\text{-}7)$$

An effort by the author to improve the fit shown in Figure 3-1 by substituting the exponential dependence of Equation (3-2) for Equation (3-1) resulted in deteriorating the fit considerably.

To determine the power exerted instantaneously in a single contraction we differentiate Equation (3-7) (See Figure 3-2):

$$P = dW/dt = 38 \times 10^{-3}/t^2 \text{ horsepower} \qquad 0.26 \leq t \qquad (3\text{-}8)$$

For the contraction of shortest possible duration, about 0.26 seconds for the previous subject, we have a maximum possible instantaneous power of 0.55 hp. It should be noted that Equation (3-8) may be in error, since in taking the derivative of fallible data one tends to raise the "noise" level. This power is a theoretical maximum of no practical value, since in 0.26 seconds, Equation (3-7) indicates no work is done. Instantaneous power over a vanishingly small time interval has no practical meaning.

Equation (3-1) was further verified in a maximal effort bicycle ergometer pedalling experiment by Dickinson(20). The bicycle had an 18cm pedal crank and a 3:8 pedal travel to wheel travel ratio. The concept of muscle shortening time becomes quite fuzzy, since Dickinson was actually

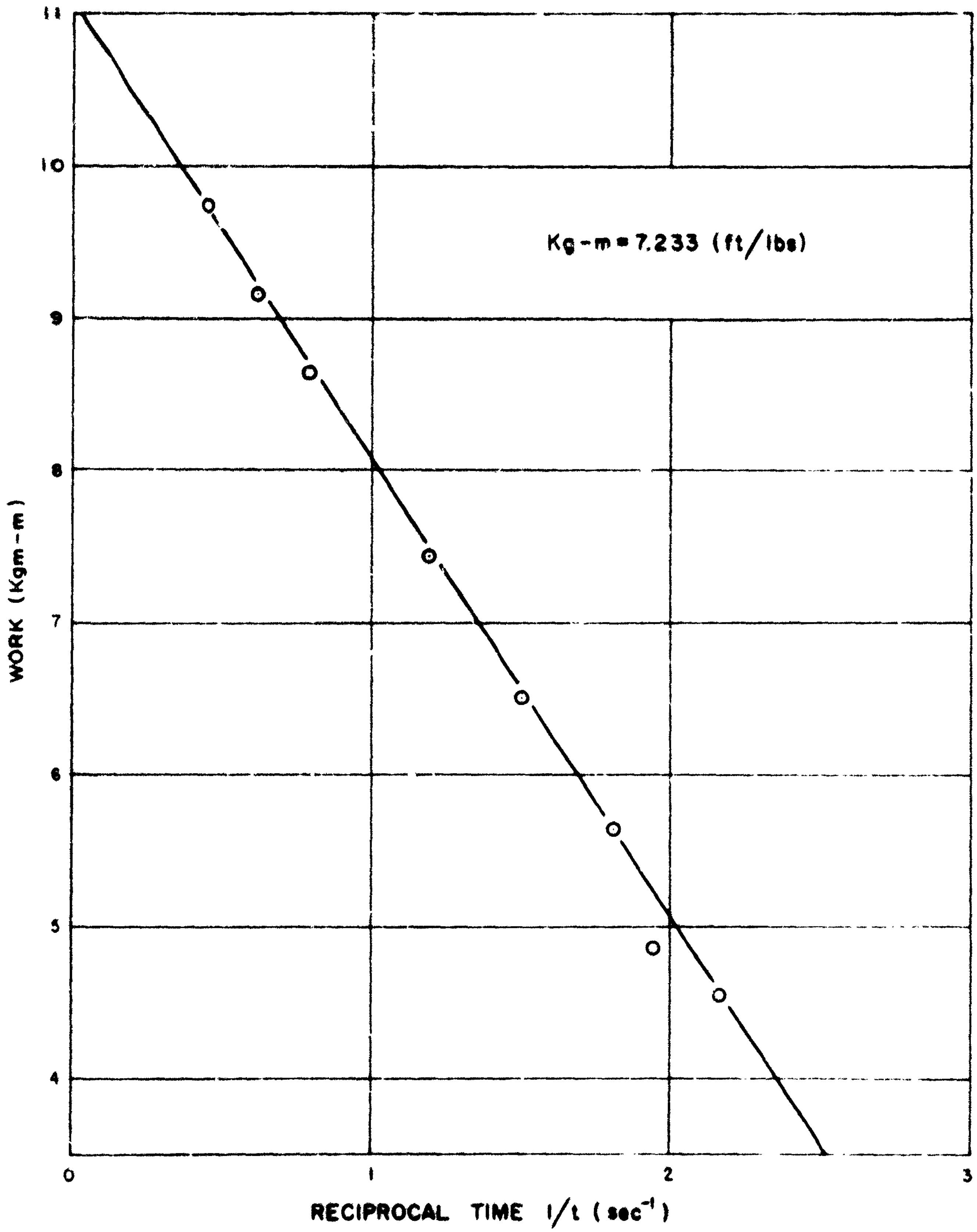


Figure 3-1. The Fit of Equation (3-7) to Lupton's Data (ref. 45)

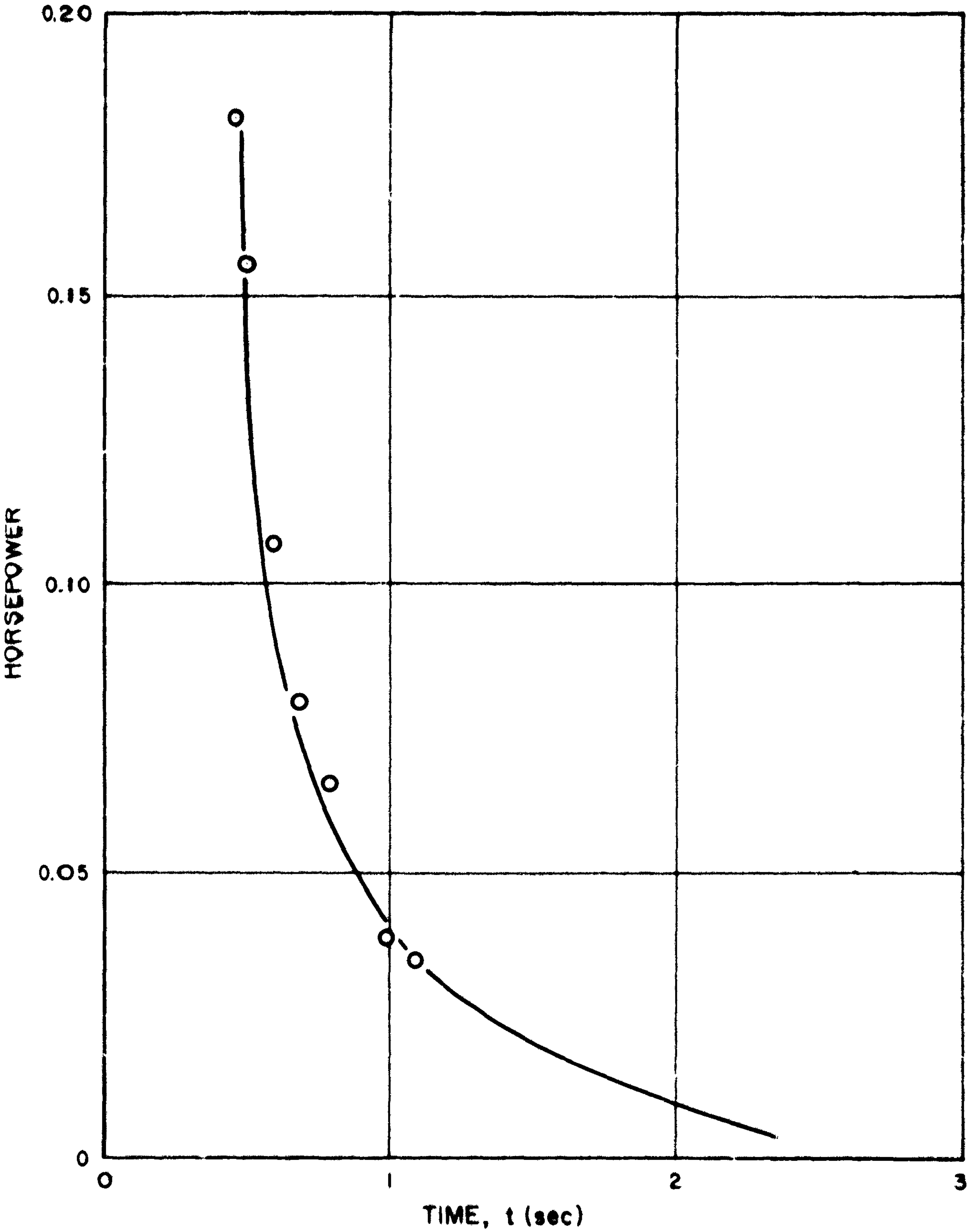


Figure 3-2 Power in a Single Muscular Contraction (from Lupton, ref. 45)

obtaining the average of a force exerted in a circular movement. The complicated muscle groupings and patterns of innervation in bicycling make the extension of Equation (3-1) from its original restricted definition questionable.

By varying the load and pedal speed, and assuming that the time for half a revolution of the pedal was equal to t, Dickinson found

$$F = F_o(1 - 0.16/t) \qquad 0.16 \leq t \qquad (3\text{-}9)$$

where for the three female subjects, 37 kg-m $< F_o <$ 56 kg-m, and for the one male subject 75 kg-m $< F_o <$ 85 kg-m. The values for k were very stable, varying from 0.159 seconds to 0.162 seconds for the four subjects studied.

Converting Equation (3-9) to the equivalent work and then instantaneous power form for the male subject:

$$P = dW/dt = 98 \times 10^{-3}/t^2 \text{ hp} \qquad 0.16 \leq t \qquad (3\text{-}10)$$

and for the average female subject the instantaneous power was:

$$P = \frac{dW}{dt} = 75 \times 10^{-3}/t^2 \text{ hp} \qquad (3\text{-}11)$$

The theoretical maximum instantaneous power in a single leg movement is 3.8 hp for the male subject, and 2.9 hp for the females. As with Equation (3-8), this maximum does not tell us much about the sustained average power over a useful duration of time.

The general form of Equation (3-1) was even verified by Lupton(45) for stair climbing exertions. One can then conclude that for single effort, maximal exertions, Equation (3-6) provides a good estimate of the relationship to be expected.

More recently, (1947), Dern, Levene, and Blair(19) claimed to have demonstrated the inadequacy of Equation (3-6), but their experiments were not convincing since the magnitude of components of muscular tension along the major axis of the muscle was not readily apparent in their complicated experimental combination of linear and rotational movements in an elbow flexion effort. They disclaimed the underlying thesis of Hill that microscopic measurements of muscle activity could substantiate theories of the behavior of large muscle groupings in the intact organism, by pointing out that

(a) a gross muscular movement was extremely complicated in terms of neural innervation.

(b) antagonists as well as agonists were active during the contraction, thus making the isolation of effects perilous, at best.

A later study by Wilkie(59), however, provided a more appropriate test of Equation (3-5), and probably is the best summary of the present status of Hill's theory. Wilkie found that the antagonists relaxed in rapid movements against external forces, and that only in a slow, moving fixation type of movement were the antagonists active.

In an elegant series of experiments on maximal elbow flexion, Wilkie demonstrated that Hill's theory as derived for isolated muscles could be shown to apply to gross muscular behavior. The only ambiguity which remains is whether the hyperbolic F versus V curve of Equation (3-5) is due to muscle characteristics or to the innervation scheduling of the muscle. Evidence from excised muscles favors the hypothesis that the hyperbolic force-velocity curve is characteristic of the muscle itself. Figure 3-3 shows a plot Wilkie obtained for elbow flexion. Each point is the mean of 30 observations and the variances are negligible. Three other subjects had similar curves. The constant, a/P_o, in Equation (3-5) was 0.20 for the subject of Figure 3-3. The range of measured a/P_o for the four other subjects was $0.2 \leq a/P_o \leq 0.48$. The experiment was such that the subject rested his upper arm on a table, and pulled a load with his hand by rotating his forearm in a vertical plane about the elbow joint.

Referring back to Equation (3-5), one can determine the condition for maximum power, FV. This occurs when $F/a = V(1 + F_o/a) - 1$. For Figure 3-3, for F = 7 dynes, V = 2m/sec, the power is 140 watts or about 0.2 hp. This figure is considerably more accurate than the previously quoted value for maximum horsepower from Equation (3-8). However, since the abscissa in Figure 3-3 shows hand, not muscle fibre velocity, the maximizing condition in Equation (3-5) does not strictly apply.

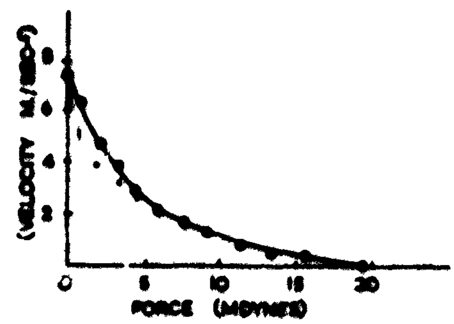


Figure 3-3

Force vs. Velocity for Maximal Effort Human Elbow Flexion (from Wilkie, ref. 60)

In plots of the velocity of the hand versus time in the maximum effort elbow flexion experiments, Wilkie noted an oscillating tendency in the response which varied with the force load on the arm. He related this to Hill's postulated contractile plus elastic element model for a muscle. Seeking to magnify the effect, he incorporated a series compliance in the cable on which the man exerted force. He was then able to indicate experimentally that the oscillation was, indeed, due to a series compliance. Wilkie then postulated the following simple model for a muscle (see Figure 3-4).

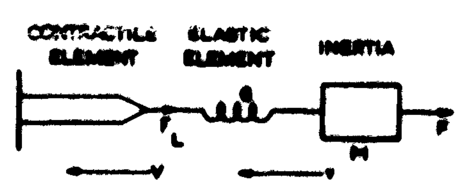


Figure 3-4. Diagram of Muscle Pulling Against a Load (from Wilkie, ref. 60)

In Figure 3-4, the symbols represent the following:

F_L = isotonic load force against which the arm is pulling
F = tension in the muscle
v = the velocity of the hand
V = the velocity of shortening of the muscle
G = the compliance of the muscle structure (assuming no external compliances)
M = mass of the forearm and associated measuring apparatus

From Figure 3-4 we see that the equation for the forces is

$$F = F_L + M\, dv/dt, \tag{3-12}$$

and the equation for the velocities must be

$$V = v + G\, dF/dt. \tag{3-13}$$

Rewriting Equation (3-5) as

$$V = (F_o + a)b/(F + a) - b, \tag{3-14}$$

then eliminating F and V, we obtain the equation of motion for M:

$$V + GM \frac{d^2v}{dt^2} = \frac{(F_o + a)b}{F + M\frac{dv}{dt} + a} - b \tag{3-15}$$

The constants in Equation (3-15) were obtained as follows:

F assumed the following experimental values: 4.53; 9.63; 11.57; and 13.42 megadynes.

That portion of the mass, M, characterising the apparatus was obtainable directly. The rest of the mass, M, which represented the arm was obtained by first determing the volume of the arm by immersing it in 2-cm steps into a large container of water and measuring the overflow. The essentially continuous volume measure was needed for subsequent moment of inertia calculations. The mass of the bone was obtained by comparing an X-ray picture of the arm with bones of known mass, and the flesh mass was determined from its volume and average density. For one subject, the moment of inertia about the elbow resting on a table, was 0.53 kgcm^2, and the equivalent mass at the hand was 0.52 kg.

The constants, F_o, a, and b, were obtained by fitting curves such as the one shown in Figure 3-3.

The compliance, G, was measured from isometric contraction data. Accepting the assumption that the force-velocity relationship is the same in the contractile elements under either isometric or isotonic contraction, one can see that the elastic element whose compliance we seek must lengthen during contraction to preserve the isometric condition.

The rate of shortening of the contractile element is

$$V = \frac{(F_o + a)b}{(F_o + a)} - b ,$$

and the equalizing rate of lengthening for the elastic element is

$$G \frac{dF}{dt} .$$

Equating and solving for G:

$$G = \frac{(F_o + a)b}{F + a} - b \frac{dF}{dt} . \qquad (3\text{-}16)$$

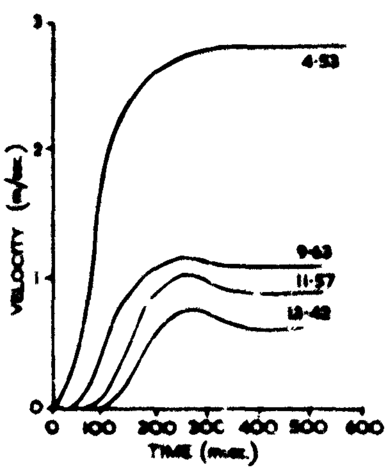
Consequently, G is determinable from the F vs. t curves and from known constants of the muscle system.

Since the computation of the series compliance assumed it was all localized at the hand, the values are biased upward. In addition, G was a decreasing function of F. The values of G selected correspond to 0.67 F_o, and for the five subjects they were as follows:

Subject	A	B	C	D	E
G (cm/megadyne):	1.1	1.1	0.6	3.6	0.5

Since Equation (3-15) was not integrable directly, it was solved by means of an analogue computer(60). Figures 3-5 and 3-6 show the similarity between the measured and the computed velocity vs. time curves.

Velocity-Time Curves (from Wilkie, ref. 60)



Velocity-time curves obtained electrically compared with those found experimentally in muscle.

N B —The calculated curves have not been fitted to the experimental ones. All the constants required for solution were determined independently

Ordinate Velocity of hand in metres sec

Abscissa Time in m.sec.

The number appended to each curve indicates the tension F in dynes x 10^4 against which the movement was made

The curves have been displaced from their common origin. The bar which ends each experimental curve marks the anatomical limit of the movement. In the faster movements there is not enough time for the maximum velocity to be reached

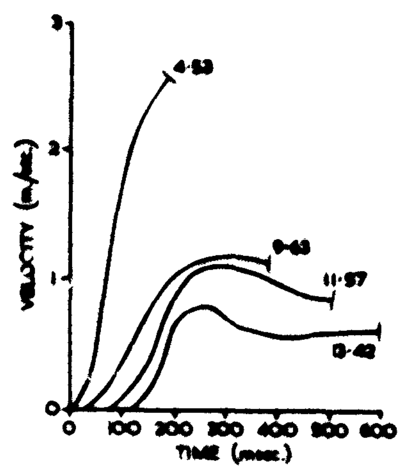


Figure 3-5. Obtained Electrically

Figure 3-6. Obtained Experimentally

The foregoing has related to methods for measuring and describing short bursts of human power output. In general, the human was interacting directly with a simple mechanism during these maximum effort exertions.

There are, however, many interesting examples where a man does work on his own body by propelling it forward, thus imparting either kinetic energy or both kinetic and potential energy to his body, depending on the

direction of his motion. In the following pages we will see how such measurements can be made for the case of running. These data are not of direct value in the design of machines but they do illustrate some of the characteristics of the human prime mover.

In an effort to substantiate Hill's early theory of internal viscosity effects in muscular exertion for an experimental study of sprint running, Furusawa, Hill, and Parkinson(28) proceeded as follows:
Let M be the mass of a runner who can exert a maximum force, F, in propelling himself forward horizontally on a level track. Assume

$$F = fMg, \tag{3-17}$$

where g is the acceleration of gravity and f is a dimensionless characteristic of the runner's build, strength, fitness, etc., which has the following range: $0.5 \leq f \leq 1$. Internal losses and fatigue put an eventual limit on the velocity which the runner can attain, but Hill assumed that his hypothesized viscous drag effect acted first. This viscous resistance may be assumed proportional to muscle size and roughly proportional to body weight as follows:

$$\text{viscous losses} = M\ v/a, \tag{3-18}$$

where, M is the runner's mass, v is the horizontal forward velocity, and a is a constant.
The differential equation of a runner, up to the stage where fatigue begins, is therefore:

$$M \frac{d^2y}{dt^2} = fMg - \frac{M}{a}\frac{dy}{dt} \tag{3-19}$$

The solution is

$$y = fga\left[f - a\left(1 - e^{-t/a}\right)\right],$$

and

$$v = fga\left(1 - e^{-t/a}\right) ; \qquad (3\text{-}20)$$

where y is the distance run, and t is the time measured from the moment that the muscles have become innervated.

A series of experiments were performed on sprinters and the results show a remarkably close fit to the solution of the foregoing equation so that:

$$v = v_o\left(1 - e^{-t/a}\right), \qquad (3\text{-}21)$$

where $v_o = fga$ is the terminal velocity before the onset of fatigue. The Equation (3-21) form is commonly found where an energy storage device is in the process of discharging its energy subject to resistive losses. The constant, a, is commonly designated as the time constant for this system.

Best and Partridge(4) expanded this type of study by loading their runners with a resistance, R, by means of a cord attached to the waist which unwound from a friction-loaded capstan. They found that the dynamic equation of running was modified, as expected:

$$M\frac{d^2t}{dt^2} = [fMg - R] - \frac{M}{a}\frac{dy}{dt}. \qquad (3\text{-}22)$$

It is possible to solve for F, the propelling force in the sprint equation, (3-7), by means of detailed time and velocity measurements of the runner. Using F and the runner's velocity, his generated power can be computed. The fastest sprinter studied by Furusawa, Hill, and Parkinson,H.A.R.,

weighed 165 pounds, ran 11.46 yards per second, with a force equal to 0.815 of his weight. In so doing, the sprinter developed 8.5 horsepower in overcoming both internal and external resistances.

It is important to note again that the conventional measures of power developed by a man generally derive from a measure of some useful output rather than from the basic relationship:

$$\text{Power} = \bar{F} \cdot \bar{v},$$

where F is the force along the velocity vector, v. These indirect and conventional work or power measures often take the form of simply measuring the changes in potential energy in the earth's gravitational field. Actually, of course, the man can do work in creating kinetic energy as well, although such work is often ignored since it is rarely useful in a power sense. Thus, the work done in climbing stairs is the sum of the kinetic energy due to the velocity achieved, which energy is dissipated in heat on coming to a stop, and the potential energy in the gravitational field. The potential energy is the only convertible energy remaining; hence, it is the commonly measured quantity. This ambiguity in measurement techniques, together with the fact that the power is dissipated both internally and externally, is what makes the 8.5-hp figure appear outlandish.

The viscous friction model of Hill, although not tenable on a muscle fibre level, does serve to provide an adequate average description for the gross data of running. Fenn made a detailed analysis of running to determine where and how the energy losses actually occurred rather than assuming the presence of some average viscous resistance. His technique consisted of using motion pictures for determinations of the motion of the body's center of gravity and of the limbs about the center of gravity, as well as a recording platform to measure the forces developed during the forward and backward pushes in the stride(22,26).

For a runner developing 13 hp, as measured by his total oxygen consumption, of which 7.8 hp represents recovery energy costs, Fenn determined that 2.95 hp were accountable as external work:

Work against gravity	0.1 hp
Velocity changes of body	0.34 hp
Acceleration of limbs	1.68 hp
Deceleration of limbs	0.67 hp
Overcoming wind resistance	0.16 hp
	2.95 hp

Facial and body muscle contractions, sideways motions of the body, and work against viscosity are not included in this breakdown. The remaining 2.25 hp is divided between fixation energy, waste heat, and internal frictional losses. The energy remaining for viscous friction losses, in contradiction with Hill's theory, is thus quite small, since the other two losses are far from negligible. The horsepower for Fenn's example comparable to Hill's 8.5 hp figure is 2.25 hp + 2.95 hp = 5.2 hp. Evidently Fenn's sprinter was weaker than Hill's.

Thus, although there is no question but that Fenn's refined analysis of sprint running is definitive, the formal analogy between Equation (3-20) for all its approximate nature, with an electrical resistance and inductance circuit is still enticing. The velocity, v, is the analog of the current, i, in an RL circuit whose transient behavior for a step voltage is being studied. This simple analogy is of significance in a hoped for impedance-matching analog of man power transmission.

In such an impedance-matching model, the ability of the muscle to convert kinetic energy to potential energy and then to release this energy at an appropriate time is of importance. A muscle which takes up the impact, Ft, of a moving limb stretches and may be thought to have acquired potential energy. The stretched muscle, however, must maintain continuous contractions in order to retain this potential energy. Thus, the muscle is continuously losing and redeveloping energy. A muscle may

be thought of as "charging storage" for stored energy at a rate determined by the balance of the dissipative and regenerative actions. In the movements in running, the maintenance cost or "storage charge" is less than the cost of redeveloping the tension in the muscles(23,27,61). In other movements, this may not be the case. In any event, a simplified spring constant picture of muscle energy storage in isometric contractions, is at best, a rough approximation to a complex process, but if it is possible to measure equivalent attenuation factors and effective spring constants, it is not an unworkable concept for gross calculations.

SECTION IV

EFFICIENCY OF MUSCULAR WORK

Although there are many maximum effort exertions in which power may be generated without regard to cost, there is considerable utility in knowing how efficiently the human body converts its stored fuel into mechanical work. Such information would be of direct value in establishing caloric requirements for the performance of energy-consuming tasks such as marching, industrial work, athletics, etc.(21,47). A more exciting possibility, however, is implied in the projected encapsulation of human operators for future space travel. Such an encapsulation would require a knowledge of the heat transfer and mechanical properties of the man in this closed system. Since fuel would be at a premium, it might not be feasible to use as many power-amplifying devices as we employ under our usual conditions of existence. In such a case, a human passenger would have to pay in services for his weight by observing, controlling by perhaps changing the vehicle's attitude, and, when necessary and possible, generating power. The human's capacity to do work is controlled by his fuel, the amount of oxygen available to him, and by the carbon dioxide concentration in his ambient atmosphere. The logistic economy involved in supplying food energy to the human would present a painstaking though solvable problem. An associated problem is that the human can live only in a rather restricted temperature range. It is necessary, consequently, to consider the heat balance problems occurring in this isolated system where the human operator may be called on to do work. Actually, muscular activity is the major means available to the human for controlling his body temperature.

Muscle activity produces heat during the mechanical response - so-called "initial heat", and during the reenergizing or recovery period - so-called "recovery heat." The initial heat is composed of two parts: "heat of activation", A, and "heat of shortening", ax, where x is the distance of shortening. The heat of shortening is proportional to the

amount of shortening by the constant, a; the heat of activation is presumably the waste heat of the chemical reactions by which the transition from rest to activity occurs. This heat is unaffected by shortening or the performance of work, and it is at its maximum rate before the mechanical response is detectable, and vanishes at the onset of relaxation. Heat of maintenance is similar to heat of activation and is a summed effect, after internal shortening is completed, of the various heats of activation characterizing the establishment of a state of full activity(38).

The heat of recovery is the exothermic balance of a series of endothermic and exothermic reactions which accompany the oxidation processes by which the muscles slowly restore the chemical substance from which the muscular activity derives its energy. This recovery heat, in the presence of oxygen, is approximately equal to the total initial energy released as activation heat, shortening heat, and work. Thus, the energy of contraction, E, is

$$E = A + ax + W, \tag{4-1}$$

and the total energy output is approximately 2E. The efficiency of performing would then be

$$Eff = W/2E \tag{4-2}$$

As a practical matter, the total energy output of the body is generally measured either by means of a huge calorimeter, or, more commonly, from the oxygen consumption as measured by a Douglas bag or similar device. The oxygen consumption can be related to the body's energy output from the fact that the amount of heat liberated in the oxidation of food in the body is the same as the amount of heat liberated upon burning the same food external to the body in a calorimeter. Consequently, measuring the oxygen necessary to oxidize different foods externally, and the heat developed, we

can get a measure of the heat developed by a human body while consuming a known amount of oxygen and burning certain types of fuels. The respiratory quotient, which is the ratio of CO_2 liberated to O_2 absorbed, is an index to the type of fuel the body is burning. Tables have been prepared which relate O_2 consumption to heat output for various respiratory quotients. The ranges of heat produced per liter of O_2 at standard temperature and pressure are from 4.686 to 5.046 kilocalories(59). The lower figure is for fats and the higher for carbohydrates. The respiratory quotient varies during the course of work such that it increases during heavy work, takes a steep short duration rise at the completion of the work, and then slowly subsides to a rest level. Thus, a conversion of O_2 consumption to kilocalories requires, at the very least, a measure of the average respiratory quotient, since the body acts in a gross way as if it were burning different fuels during exertion, than during rest(13).

Another problem in getting an accurate measure of the total energy produced arises from the fact that while recovery is rather slow, i.e., a few minutes to an hour, the contraction of the muscle which produces work may take no more than a few milliseconds or, at most, a fraction of a minute(57). The result of this is that after a sustained effort, the contraction processes acquire a free energy debt, in the thermodynamic sense, to the recovery processes. This debt is a function of the effort and duration of the work alone, and its measure is the "oxygen debt". The oxygen debt is the excess of oxygen absorbed over the normal requirements after the exertions have ceases. In maximal effort exertions, the energy reactions are anaerobic; hence, a steady state is not possible. Fatigue increases as the energy resources are depleted and as the waste products increase. With such efforts, diet, ability to dissipate heat, limiting atmospheric O_2 concentration, lung capacity, etc., become of great importance(21).

The importance of lung capacity and O_2 concentration arises from the fact that as exertions approach a maximum effort, the depth and rate

of respiration increase so that the body is able to absorb oxygen at about 20 times the resting rate. This oxygen is needed to oxidize the waste products of muscular activity. It is the oxygen supply, not the inherent muscular strength, which limits the human's power production.

Over short periods of time the vascular system can adjust so as to allow a greater amount of oxygen to be supplied to the hungry active muscles at the expense of other parts of the body; such as the viscera. In this way, peaks of power are achieved essentially by spreading the oxygen debt over the whole body rather than limiting it to a specific muscle complex. The limiting power a man can generate without building up an oxygen debt can be determined from the maximum rate at which he can absorb oxygen from the atmosphere. This figure is about 4.5 liters per minute.

Using a crude average of the Zuntz data(62), we can convert this oxygen consumption to an energy production of about 22.5 kilocalories. Later in this section we shall see that the man's efficiency for conversion of this energy to mechanical energy is about 20%, with 25% as a possible upper limit. On this assumption, the man's peak steady-state horsepower is theoretically 0.4 to 0.5 hp. For short bursts, acting anaerobically, the man can put out a greater effort, but he must repay the oxygen bank at the end of his efforts. In Section V, we will see how close maximum-effort cycling studies come to this figure(61).

In the early twenties, Hill developed a theory, based on considerations of maximal effort contractions of individual muscles, which held that maximum efficiency should occur for a certain speed of muscular contraction. The basis of this theory was Equation (3-6),

$$W = W_o(1 - k/t),$$

and the assumption that the physiological cost or energy liberated was

$$H = A + Bt.$$

The constants, A and B, had particular significance in terms of the then current theory for isolated muscle tissue, since

$$E_{ff} = W/H = \frac{W_o(1 - k/t)}{A + Bt} \tag{4-3}$$

This expression has a maximum t_m, which is the solution of

$$t_m^2 - 2kt_m - k\ A/B = 0 \tag{4-4}$$

Since k, A, and B are all imperfectly known, this relation had little practical value. Unfortunately, many of the attempted tests of Equation (4-3) were not conducted under the assumptions underlying this equation.

Equations (4-3) and (4-4) were used as an argument that efficiency was dependent mainly on speed of movement and only remotely on the amount of work done. Dickinson and Lupton(21,44) both worked within this framework to determine the optimal time for an individual muscular contraction. Lupton determined it to be 1.3 seconds for a single movement of the leg in stair climbing, 1.36 seconds for elbow flexion; and Dickinson determined it to be 0.9 seconds for a single downward movement in pedalling. Since these were gross measurements of rather complicated muscle patterns, the applicability of the theory is dubious. However, their data do point up how impractically slow maximally efficient effort is.

The magnitude of the maximum efficiency of human muscular activity is of considerable value in a complete human factors study of a man in a closed system. The best judgement on the available data for positive

mechanical work is that 20% is a reasonable figure. Some of the early studies yield efficiencies for bicycle ergometers which are about 25% (assuming an efficiency of unity for these measuring devices). This data, however, suffer from the fact that the recovery period energy was not measured. The subject in one experiment(13) was given a ten-minute rub-down after exercising; and in another pioneering study(3) no apparent effort was made to measure energy expenditures during recovery, although considerable care was directed to getting a proper metabolic base line.

The following are representative data, which are probably all biased upwards except for the stair climbing, which seems to justify Coulomb's thoughts on the subject (mentioned in Section I):

Maximum Muscular Efficiency	Work	Source
27.9%	turning a winch with arm muscles	Reach(52)
24.4%	climbing stairs (assumed 7 min. recovery time)	Lupton(44)
26.7%	arm flexion*	Lupton(45)
24-25%	bicycle ergometer	Benedict and Cathcart(3)
24%	bicycle ergometer	Campbell, Douglas, and Hobson(13)

Although there exist a large number of bicycle ergometer studies from which efficiency measures were made, these studies are comparable only in the most approximate sense. This is because many mechanical aspects of cycling, such as height of saddle with regard to the cyclists leg length, position of saddle, length of pedal stroke, etc., must be controlled to make comparisons of bicycling efficiencies meaningful. In addition to these difficulties, base line metabolism measurements are often dubious,

* Although a ten-minute recovery time was postulated, E was computed by curve fitting to get the constants in Equation (4-3), solving Equation (4-4) for t_m and substituting t_m in Equation (4-3). Since respiration measures were among the techniques used to evaluate the constants in Equation (4-3), the cumbersome computation of the efficiency leaves much room for error.

as are certain steady-state assumptions on O_2 consumption, and the closeness of approach to maximum effort exertions. Thus, one can easily imagine how the measured optimal pedal speed in bicycling can assume such disparate values as 33 rpm, 52 rpm, and 70 rpm.

Crowden, Dickinson, and Garry(16,21,29) obtained data which appear to confirm the presence of a rather flat efficiency maximum around 33 rpm. On Figure 4-1, we present data from Asmussen, Crowden, and Dickinson(2,16,21) for the power range, 0.10 = hp = 1.40. Although the absolute values may be misleading, the general shape of the curve is probably accurate. Time durations were, in all but one case, of the order of one or two minutes. The Crowden 33 rpm data were for 6 minutes.

The Asmussen data were obtained by assuming a 4.9 conversion factor from liters of O_2 to kilocalories and using Asmussen's regression fits to his data. Since Asmussen's data were derived from a regression line for constant rpm studies, they are presented as a vertical bar on Figure 4-1.

In order to approximate the relation between efficiency and horsepower generated in continuous cycling, various bits of data were combined.

Crowden's data were obtained with a bicycle arranged for maximum comfort in terms of its mechanical features such as seat position. His study demonstrated that for the same speed of pedalling there was no variation of efficiency with the duration of effort for up to 250 seconds of work. The comparison was made between the efficiency in 250 seconds of continuous pedalling at 72 rpm as compared with ten 25-second work periods, each separated by 3 minutes of rest at the same rate of pedalling. All expired air during work and recovery was measured. However, when the same total amount of work was done at an optimal speed of 33 rpm, as compared with spurts of work at 101 rpm, the optimal speed was more efficient in the ratio of 22% to 15%.

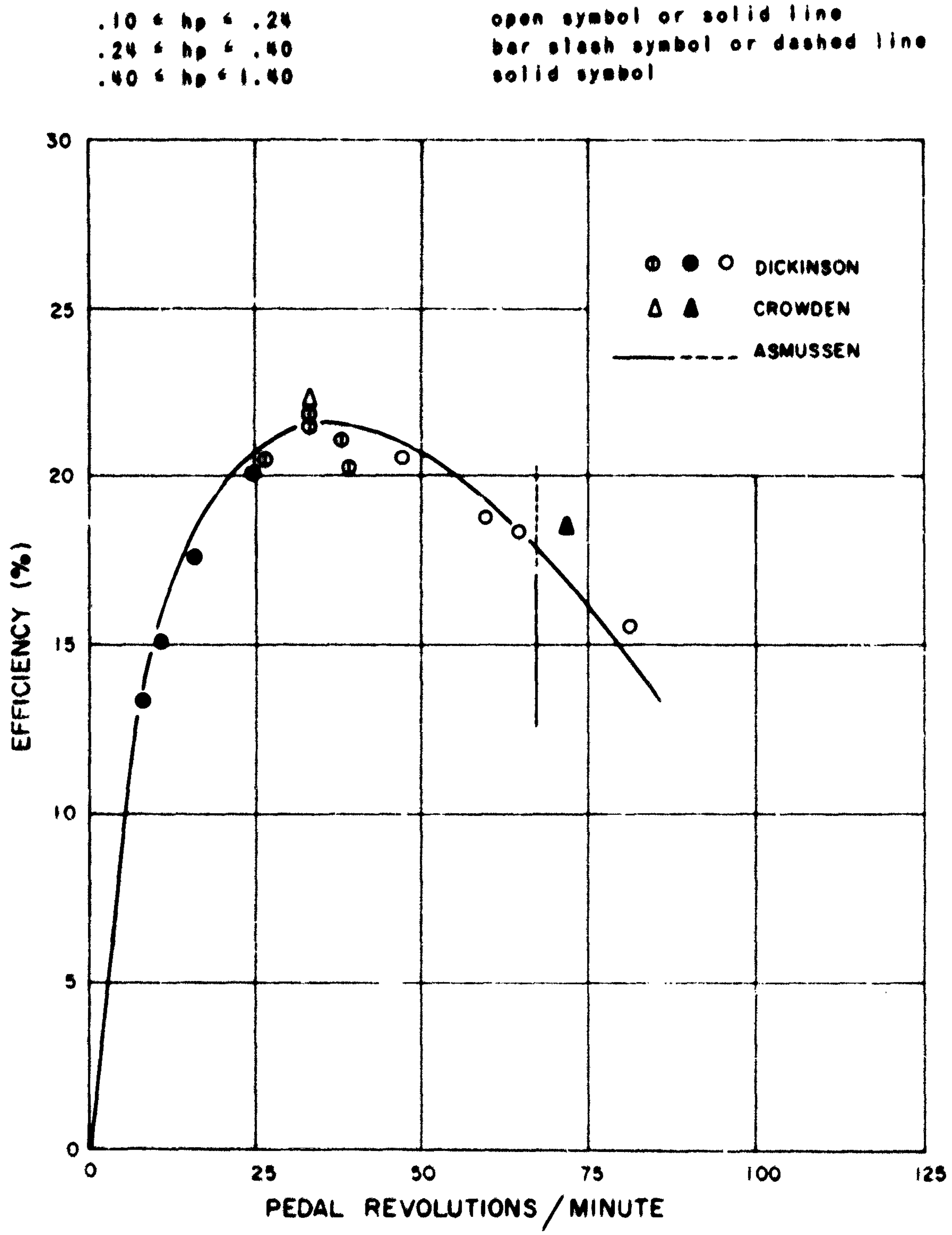


Figure 4-1. Bicycling Efficiency vs. Speed (from Asmussen, Crowden and Dickinson; refs. 2, 16, 21)

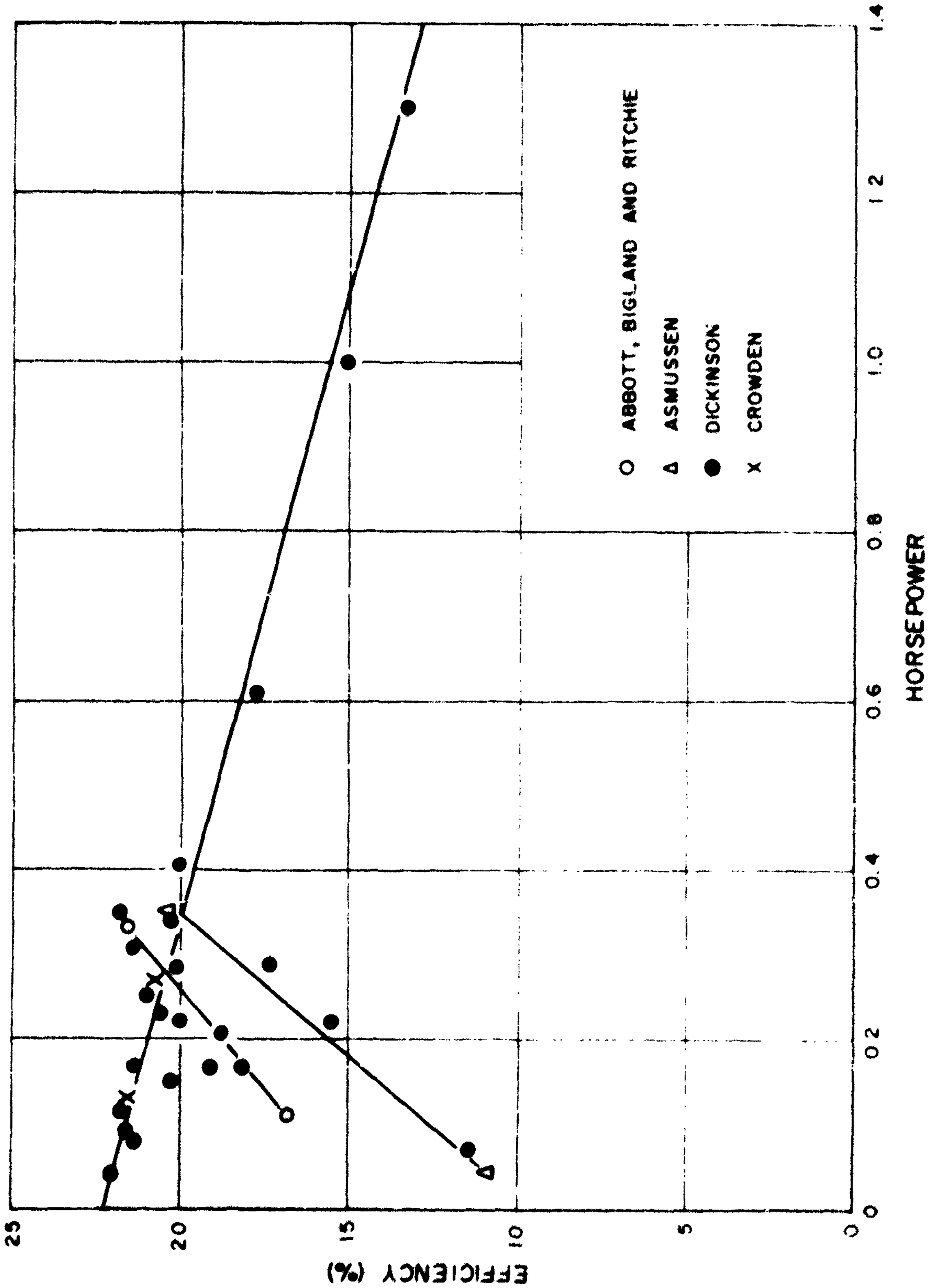
Abbott, Bigland and Ritchie(1) obtained data for varying forces, and for speeds varying from 20 to 80 rpm. The conversion of the given O_2 consumption to kilocalories of work produced is very dubious, since Abbott, Bigland, and Ritchie didn't convert the O_2 consumption to standard temperature and pressure conditions. There was no point in their doing this since they were interested in ratios of O_2 consumption in their paper.

Figure 4-2 is a plot of efficiency of horsepower generated by the bicycle ergometer without regard to velocity of motion. It will be noted that submaximum efficiencies, i.e., higher speeds, give the appearance of increasing with horsepower for a limited range of measurements. This might result from the fact that very low horsepower requirements may be unduly affected by the weight of the limbs themselves; whereas, for higher loads, the limbs are a less significant percent of the load. The clustering of high efficiencies at lower power outputs are all at about 33 rpm.

This discussion of the efficiency of positive work may be summarized by the following quotation from A. V. Hill(39):

"When positive work is done, as in climbing a staircase or pedalling a bicycle uphill - in both of which the load is approximately constant - there is a particular speed at which the physiological cost, measured in terms of energy used and oxygen consumed, is a minimum. The existence of an optimum speed depends chiefly on the balance between two opposing factors. The first one is that the quicker a muscle shortens the less is the external force it can exert, for a given degree of stimulation; hence, if the external load is fixed, the muscle, in order to shorten quickly, has to make greater effort and be stimulated more strongly - which, of course, means more energy used. The second factor, working in the opposite direction, is that the slower a muscle shortens, the longer its contraction has to be kept up in order to carry out a given extent of movement; and a longer lasting contraction means more energy used. At a certain speed the best compromise is reached and the energy used is a minimum. This region of maximum economy is rather broad, and the speeds within this maximum are not always practical."

Most of the physiological factors involved in muscular efficiency are beyond the machine designer's control. The optimal speed for efficient work may be found. This speed may not be practical, since rate of energy


ABBOTT, BIGLAND AND RITCHIE
ASMUSSEN
DICKINSON
CROWDEN
HORSEPOWER
EFFICIENCY (%)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0
5
10
15
20
25

production may be as important as efficiency for a given application.

A factor over which the machine designer has some control is the extent to which various muscle groups will be permitted to have work done on them (perform "negative" work) while lengthening, as compared with doing "positive" work while shortening. It is clearly to be expected, however, that so called "negative" work in which the muscles have work done on them as they lengthen, as in bicycling downhill, will cost less physiologically than the converse example of positive work in bicycling uphill. The converse example, however, must involve similar muscle groups. Thus, in a stair climbing exercise, the negative work study requires descending the stairs backwards, if they were ascended normally. If the limb is described by a moving fixation in that both agonists and antagonists operate, the force of gravity is cancelled; and for constant velocity, the same additional work is done with or against gravity.

The bulk of the experimental data on positive to negative work costs were obtained from bicycle ergometer studies. Asmussen's technique was to have a subject bicycle on a treadmill which could be pitched to simulate up or downhill cycling. Abbott, Bigland, and Ritchie, on the other hand, arranged two bicycle ergometers so that one subject drove the other. Thus, the man doing negative work was not back-pedalling, but was actually interchanging muscle lengthening and shortening in his work efforts. Thus, both cyclists exerted equal forces at equal speeds.

Under various conditions of lengthening and shortening the active muscles at low to moderate speeds, the ratio of positive to negative work costs measured by O_2 consumption varied linearly from 3:1 to 9:1 in Asmussen's work, for which pedal speed was held at 67.5 rpm. Abbott, Bigland, and Ritchie found the same general linear dependence on speed of pedalling, measuring a range of from 2.4:1 to 5.2:1 for speeds from 25 rpm to 52 rpm with loads designed so as to avoid an oxygen debt. At very high speeds of shortening, occurring at a bicycling speed of 480 rpm, Asmussen's measured cost ratio reached 125:1. The characteristics of very high-speed negative work is that

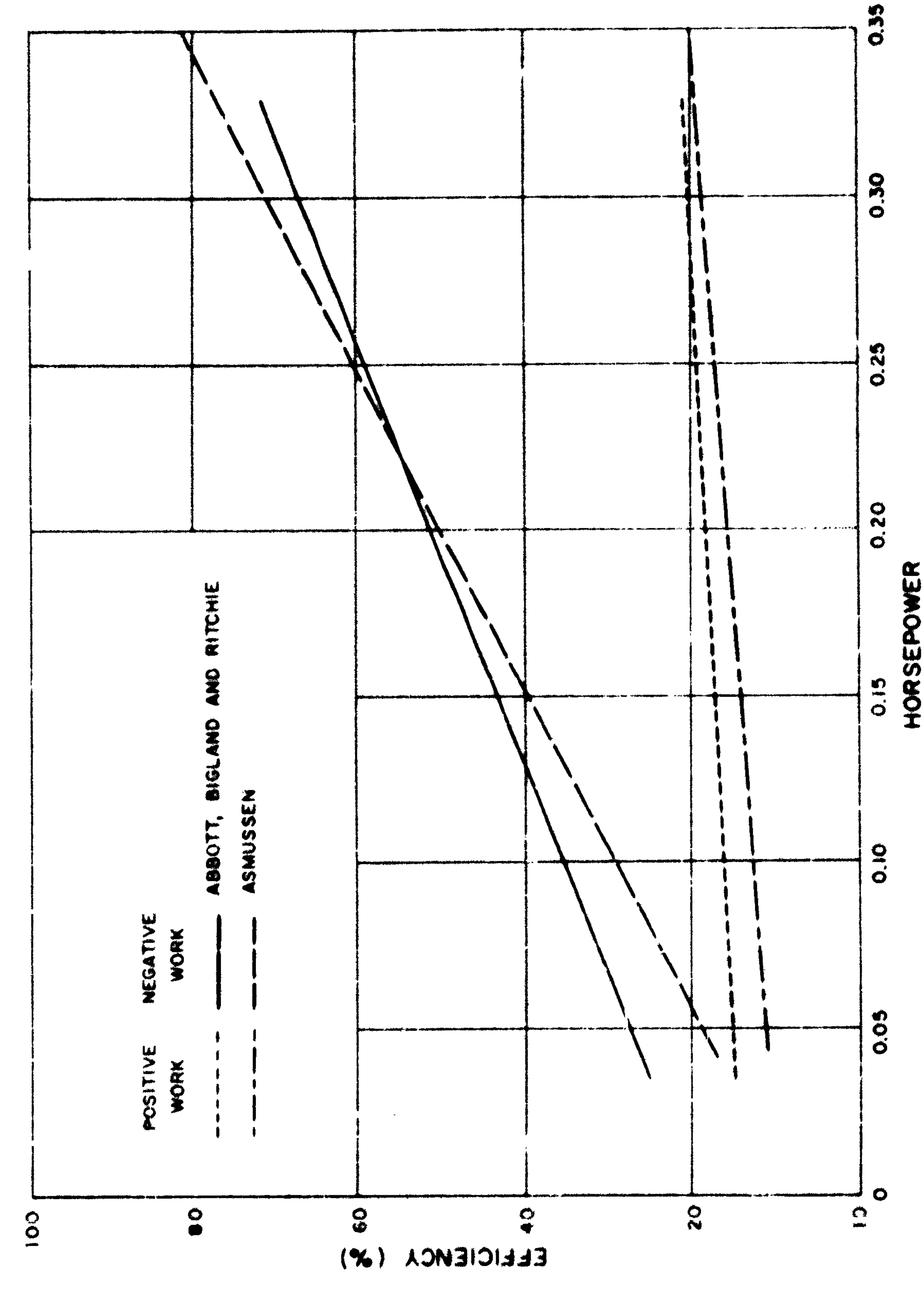


Figure 4-3. Efficiency of Positive and Negative Work (from Asmussen, ref. 2; and Abbott, Bigland, and Ritchie, ref. 1)

its cost approaches zero, and it is independent of the intensity of work over most of the range investigated.

On Figure 4-3, we have plotted the efficiencies for both negative and positive work derived from the Asmussen, and the Abbott, Bigland, and Ritchie data. The consistency in these curves is surprising, considering the questionable manipulations to which the original data were subjected.

The increased cost of positive work over negative work was explained by Abbott, Bigland, and Ritchie in the following fashion:

Consider Figure 4-4, which is a force-velocity diagram for contraction of maximum effort for human arm muscle as measured by Wilkie and presented by Abbott, Bigland, and Ritchie. Let us assume that Figure 4-4 is a description of a single muscle fiber.

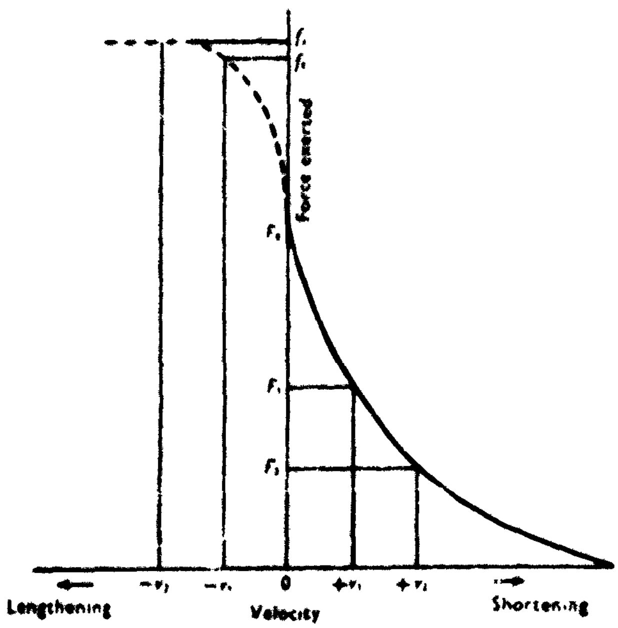


Figure 4-4. Force Velocity Diagram for Human Arm Muscle (from Abbott, Bigland, and Ritchie, ref. 1)

One can see that the force exerted by a fiber while lengthening at a given velocity is considerably greater than the force exerted by this fiber were it shortening at the same velocity. In order for the positive working cyclist and negative working cyclist to operate in equilibirum by generating equal forces, the forward cyclist doing positive work must, if the excitation frequencies for positive and negative work are equal, have more fibers in action than the resisting cyclist. At a velocity, v_1, the forward cyclist must have f_1/F_1 more fibers actuated. Since O_2 consumption is related to the number of active fibers, positive work is clearly more costly than negative work.

The increase of the ratio at high velocities may be explained by the fact that the cycling effort was maintained at a submaximal effort to avoid oxygen debt problems. Consequently, F, the load, was kept small for high velocities of pedalling. Since v increases, f increases, and f/F will also increase because of the restriction on F.

This simple increase of effort indicated by the model based on the increase in the number of fibers participating in the contraction is an oversimplification. In living animal tissue, tension below the maximum is characterised by the asynchronous excitation of many fibers at frequencies below that which would produce a fused tetanus. The economy, in terms of work output and heat liberated in a contraction, increases with the frequency of stimulation up to fused tetanus. Consequently, in a single fiber, the heat liberated, therefore the O_2 consumed in maintaining the contraction, increases less rapidly with an increase of stimulus of excitation than does the mean tension of the muscle.

Consequently, the forward pedalling cyclist's effort is measured by O_2 consumption, but not in a simple fashion because of the simultaneous increase in economy of contraction as more fibers are recruited with higher frequencies of innervation.

The resisting effort is reduced both for the simple reason given previously and/or because of the lower excitation froquency required to activate an adequate number of muscle fibers.

SECTION V

USEFUL HUMAN POWER OUTPUT

In this section it will be necessary to combine and compare data from various sources in order to achieve a consistent picture of human power production. Handwheel or crank, pedal, and whole-body-involved work will each be treated in order. Within each grouping an attempt will be made to discuss power output in terms of continuous maximum effort work, maximum effort work with rest pauses, self-paced work, and power generated over relatively short intervals. None of the foregoing groupings constitute rigid categories and the working definitions will become clear from each specific usage. Furthermore, there will be instances where it will be necessary to deviate slightly from the foregoing organization.

The Bilodeaux(5-9), singly and together, carried out a series of experiments which appear to have been generated by the observation that performance while either working or learning was characterized by decrements in response as a result of continued responding. This formal similarity gave promise that learning theory concepts might apply to repetitive motor tasks.

A Hullian scheme was used to structure the effects of rest on the inhibition of the rate of response, and the analog of the familiar conditioning finding, spontaneous recovery, was demonstrated and measured.

The cranking apparatus was a crank handle rotating in a horizontal plane with a radius of rotation of 4.5 inches. Different resistive loadings were provided by the braking force of a tachometer generator. For any given loading, the power generated was a gently accelerated function of rate of rotation. The general experimental procedure was for the subject to stand facing the crank, and holding it with a standard grip, to rotate it as fast as possible. The power generated could be measured from the revolutions over 5-, 10-, or 20-second intervals. In Figure 5-1, we present a typical performance decrement curve obtained in a maximum effort cranking. Each

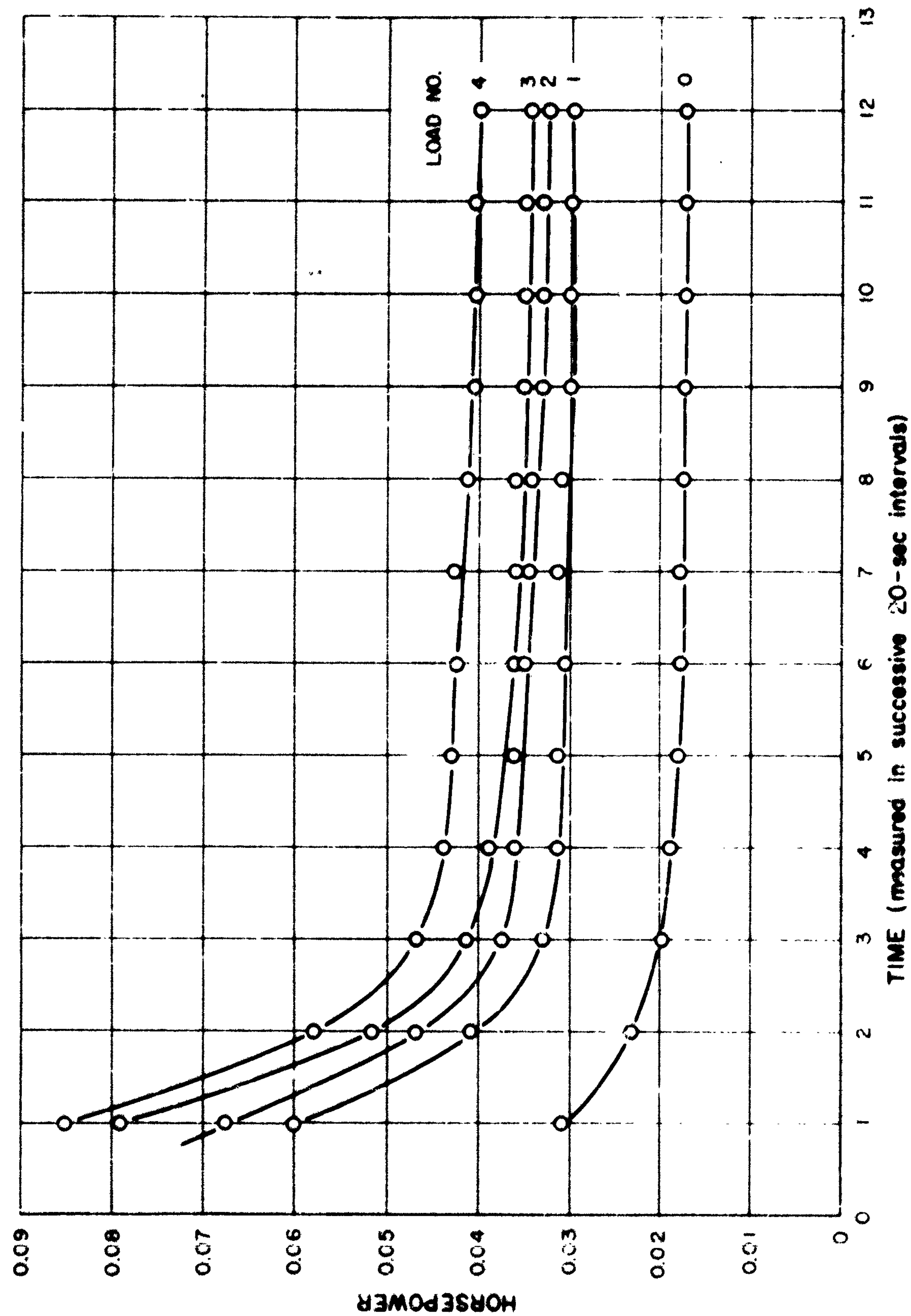


Figure 5-1. Power Generated in Hand Cranking vs. Time for 5 Resistive Loads (from Bilodeau and Bilodeau, ref. 9)

data point is a combination of 50 subjects averaged over 20-second intervals(9). Figure 5-2a represents the same data as Figure 5-1 plotted on a log scale versus reciprocal time. The general form,

$$hp = Le^{m/t} \quad 0 \leq t \leq 300 \text{ sec}, \qquad (5\text{-}1)$$

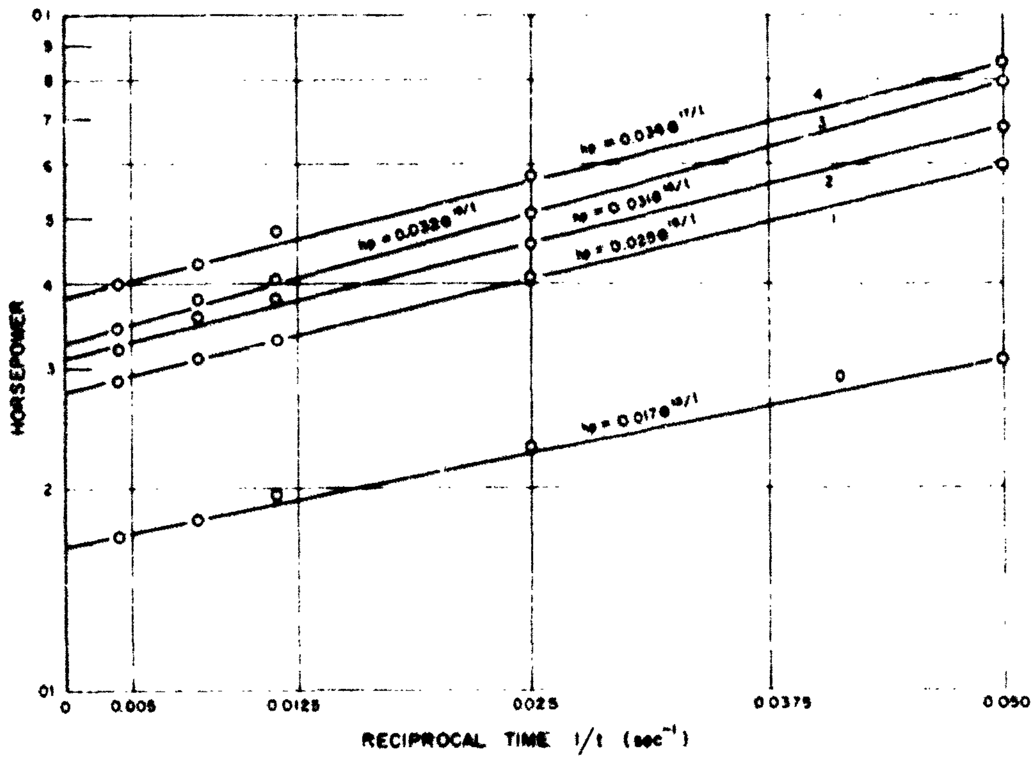
is seen to be a reasonable approximation.

The constant, L, appears to be related to the resistive torque load, and m may be a function of the musculature involved in generating the power. For example, when the shoulder and upper arm are more heavily involved, m would be expected to change.

An experiment was conducted to determine the effects of self-pacing in work generation, by varying the subject's prior work experience and his instruction so as to control the subject's expectations as to the duration of his task(8). On Figure 5-2b, there is plotted data for two groups of 40 subjects each. The first group had 30 seconds prior cranking and the second group had 3 minutes prior cranking experience with the apparatus. The subjects were led to believe they would only have to crank for 10 seconds, but instead their tasks lasted two minutes. Each point represents 40 subjects who were cranking against a load slightly heavier than load No. 4, which is a parameter in Figure 5-1. It will be noted that these data are fitted reasonably well by curves of the form of Equation (5-1). Other groups of subjects in this experiment were instructed to expect 3- and 5-minute cranking sessions, and they, too, only cranked for 2 minutes. Presumably, these men paced their effort to last for the longer expected run. Figure 5-3 is a plot on log-log coordinates of horsepower versus time for these men. As before, each point represents 40 men, and the pretrial practice was for either 30 seconds or for 3 minutes. These data are adequately fitted by the form,

$$hp = Kt^{-k}. \qquad (5\text{-}2)$$

(a) from Bilodeau and Bilodeau, ref. 9



(b) from Bilodeau, ref. 8

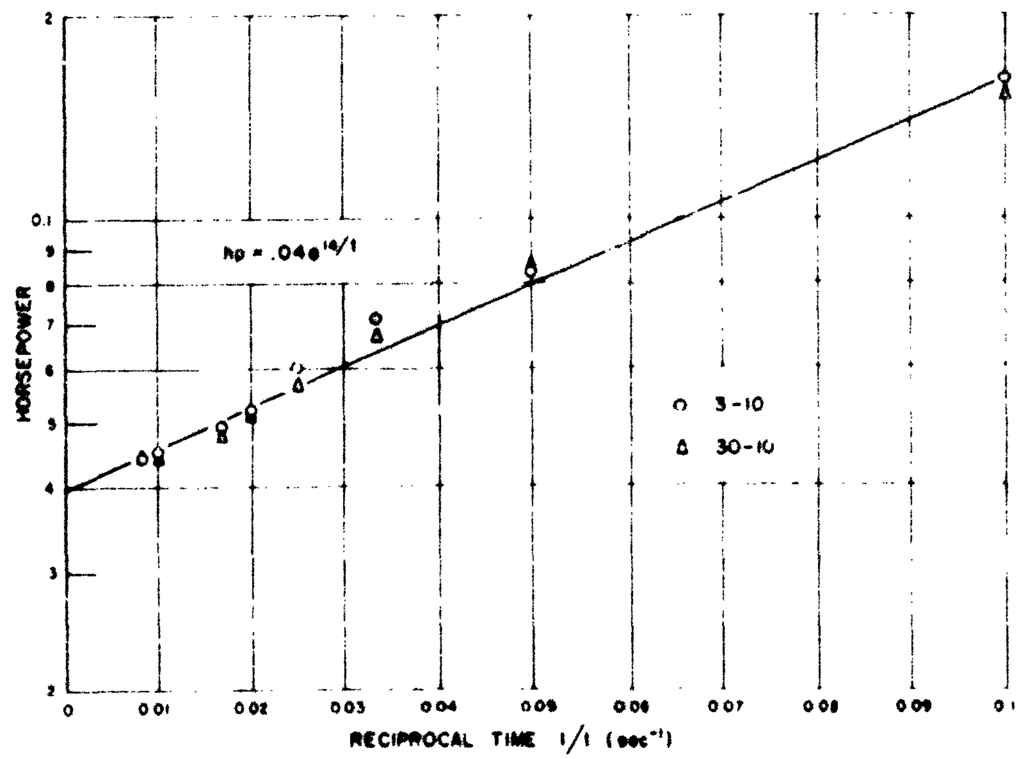


Figure 5-2. Power Generated in Hand Cranking vs. Reciprocal Time

In Figures 5-2b and 5-3, the notation describing the subject's cranking experience and expectations is as follows:

Cranking Experience	Expected Cranking Duration	Code
3 min	5 min	3-5
3 min	3 min	3-3
3 min	10 sec	3-10
30 sec	5 min	30-5
30 sec	3 min	30-3
30 sec	10 sec	30-10

Curve (- - - - -) on Figure 5-3 represents cranking experience and expectations 3-10 and 30-10 as shown on Figure 5-2b.

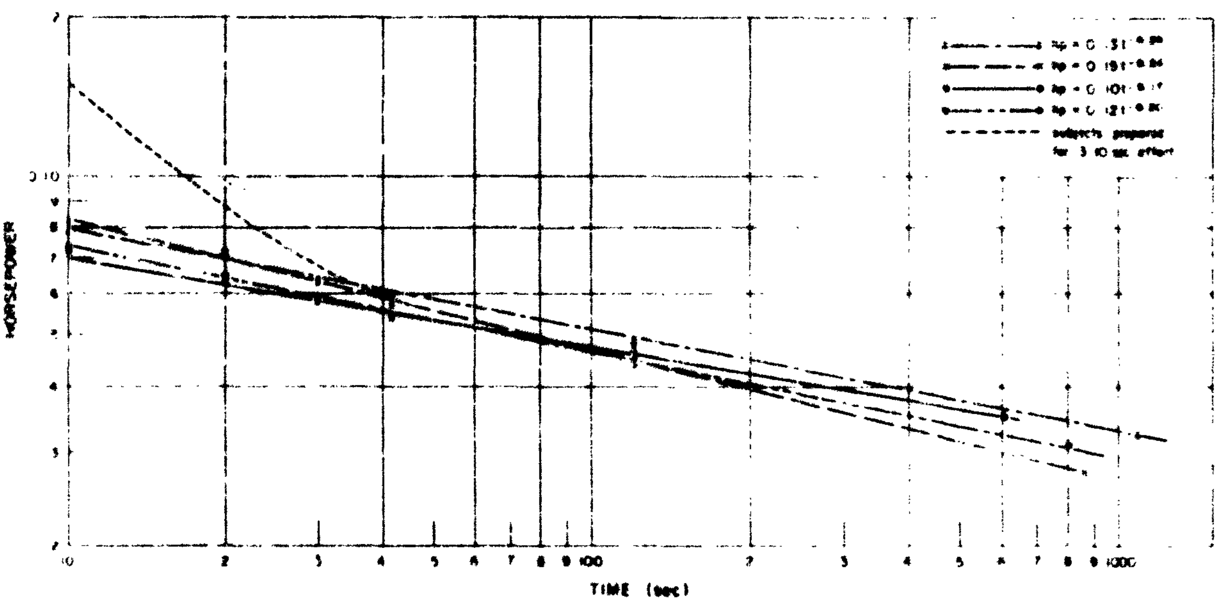


Figure 5-3. Power Generated in Hand Cranking as Influenced by Subjects' Expectations of Task Length (from Bilodeau, ref. 8)

The curves of Figure 5-2b are plotted on Figure 5-3, and they clearly are of a different form, although the stable value of power generated over a long interval of time approaches the same value in either the paced or sprint conditions. The form of Equation (5-2) is not fitted for the 10-second point for subjects not having had prior cranking experience and some concurrent notion of its duration. Figure 5-2a may not show this effect because of the 20-second time resolution.

Since sprinting and self-pacing approach the same steady-state-power-output value, one could convert from self-paced power output to sprint output for a given cranking task, for which m in Equation (5-1) is known. Consider self-paced, cranking-produced power, hp_1, generated for a time, t_1, which is 100 seconds or more. From Equation (5-1) and our steady-state assumption, we can compute the horsepower generated in a sprint of 20 seconds, hp_{20}, as follows:

$$hp_{20} = hp_1 \frac{e^{m/20}}{e^{m/t_1}} . \qquad (5\text{-}3)$$

For large values of t_1, this becomes:

$$hp_{20} \cong hp_1 \, e^{m/20} \qquad (5\text{-}4)$$

The constant, m, must be obtained from a fitting obtained to data obtained over equivalent conditions.

At this point, it is well to mention that neither Equations (5-1) nor (5-2) can be expressed as solutions of any simple differential equation. This shortcoming means that these curve fits are descriptive conveniences rather than techniques for obtaining a fresh insight into a process.

In Section II, the question of the possible existence of an optimal rest period was raised. In view of the demonstrated recovery of power producing capacity after working, one might seek that rest period which yielded an average power, over a given time interval including rest pauses, which was greater than the power averaged over an equal interval of continuous work.

Bilodeau(7) studied interpolated rest periods in a cranking test using their heaviest resistive loading on the crank.

Five groups of 54 basic-trainee airmen each cranked as fast as possible. Each group performed ten 30-second trials, and the groups differed in that rest periods of 0, 10, 30, 90, or 180 seconds were interpolated after each trial for each different group. Data on power generated were obtained over 10-second intervals of the work trials. Figure 5-4 illustrates the recovery of power output following rest.

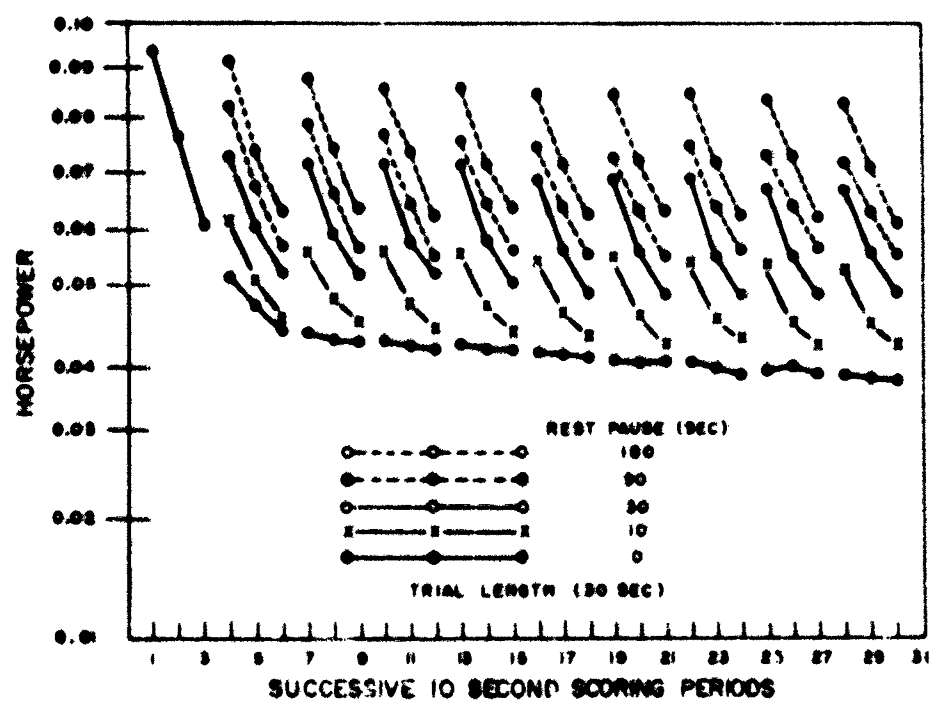


Figure 5-4. Mean Rate of Response vs. Successive Scoring Periods in a Repetitive Motor Task (from Bilodeau, ref. 7)

Computing the average power generated over the first five minutes for each condition, and plotting this against the percentage of time worked, we obtain Figure 5-5. In these particular circumstances, continuous work is clearly most productive.

A study of what was essentially a self-paced power-generating task was conducted by Katchmar(40). The subjects, who were military personnel, stood facing a crank which they rotated so that the plane of rotation was perpendicular to the horizontal and the axis of rotation was in the direction that the subject faced. Crank radii of 4, 5, and 7 inches were available, and constant torques from 0 to 90 in-lbs could be produced. The instructions were ambiguous, but they appear to have been interpreted as "crank at the fastest rate which you can maintain for the longest period of time". The subjects apparently stopped when they could no longer maintain their selected rate of turning. In all cases, time was called at ten minutes.

Figures 5-6a and 5-6b present plots of horsepower generated vs. time the self-paced task was maintained. Each data point is the average of 5 subjects. These curves were fitted, disregarding the experimentally imposed 10-minute point.

The analytic approximations determined were:

Cranking Radius (in.)	Approximating Function	
4	$hp = 5.0\ t^{-.88}$	
5	$hp = 3.0\ t^{-1.2}$	$60 \le t \le 600$ seconds
7	$hp = 0.14 + 17 \times 10^{-5} t$	

The incongruity of the horsepower versus time function for the 7-inch cranking radius is difficult to understand. The 4- and 5-inch radius cranks produce data which indicate that the subjects were interpreting these tasks as self-paced by comparison with Figure 5-3.

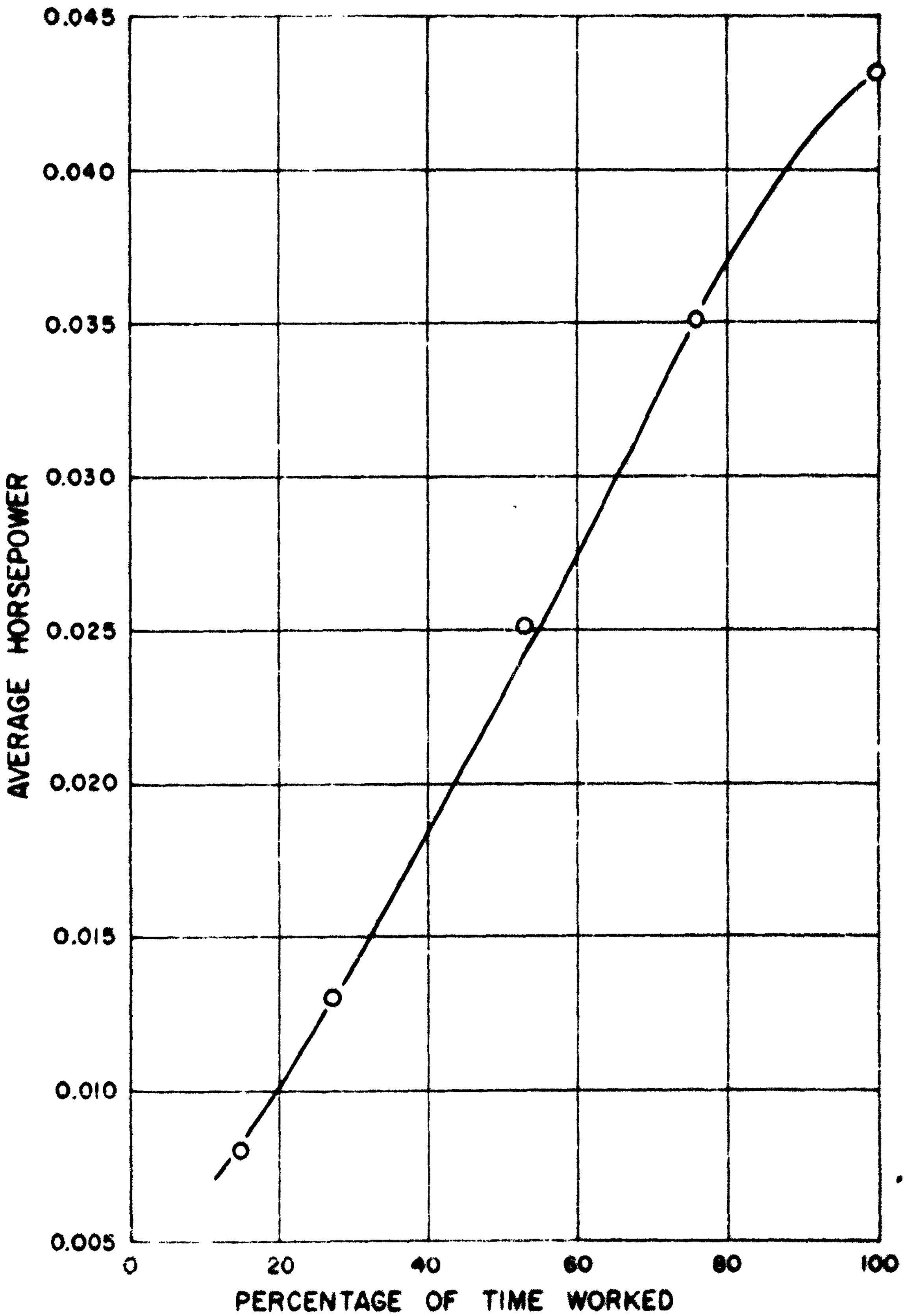


Figure 5-5. Average Horsepower vs. Percentage of Time Worked (from Bilodeau, ref. 7)

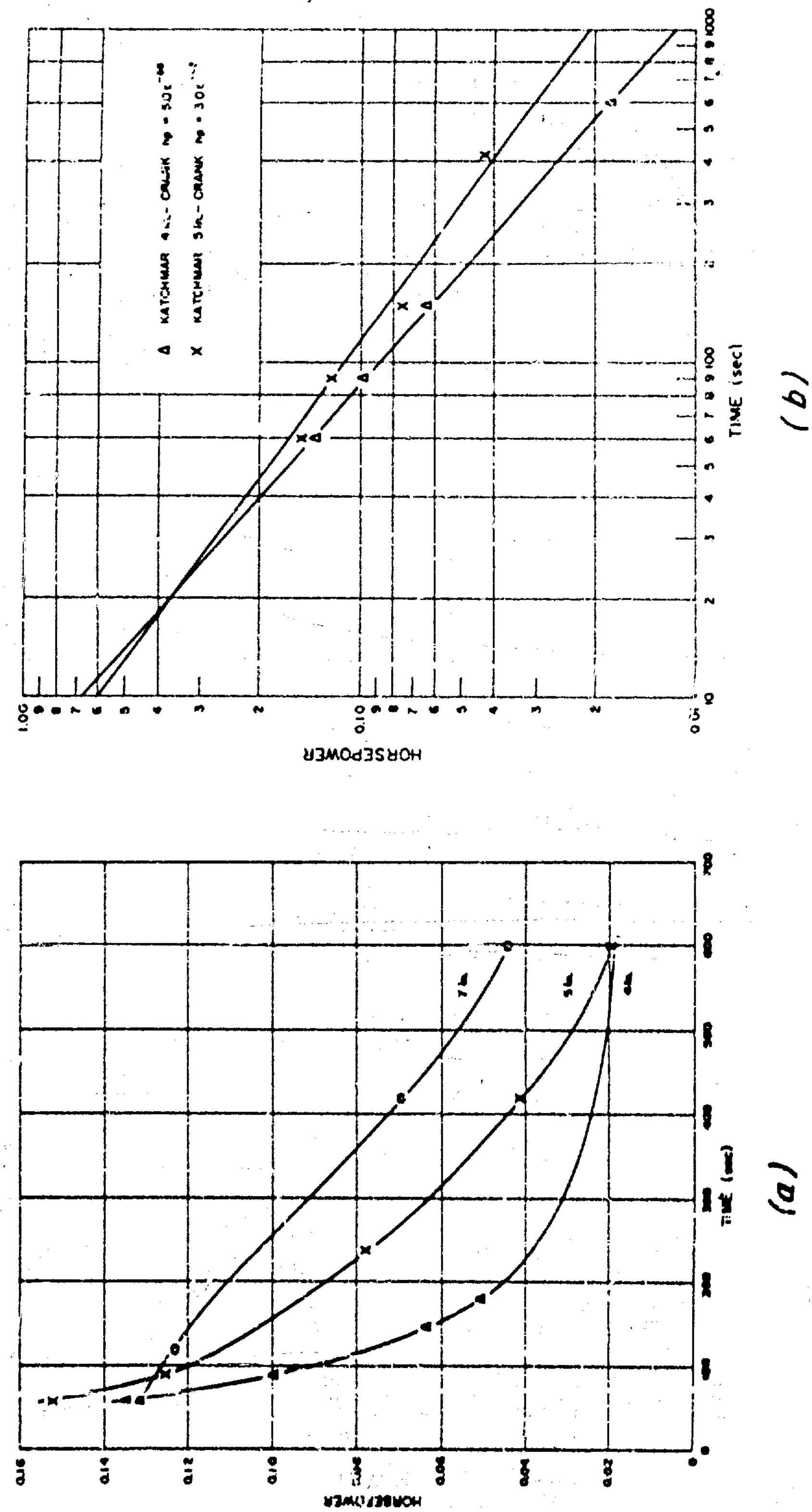


Figure 5-6. Self-Paced Power Generation (from Katchmar, ref. 40)

One can plot average power generated over intervals from 1 to 10 minutes in this self-paced task against the tangential resistive force on the crank. Figure 5-7 shows this relationship for the three cranks. It is conventional to measure torque in problems of rotational dynamics, where hp = $T\omega$, where T is torque, and ω is angular velocity. Since we would like to separate the interactions of maximum speed of rotation (a skeletal and muscular effect), as well as to maintain radius of cranking as a distinguishable parameter, the abscissa of Figure 5-7 is expressed in tangential force rather than as torque. Torque equals the radius of cranking multiplied by this force.

In the following we will discuss the generation of power by cranking over intervals of time which are in most cases, less than 30 seconds.

Hick and Clarke(33) studied the rotation of cranks of 4.25-inch radius of rotation in both one- and two-handed control in a tracking task. The subjects were seated with the crank's plane of rotation vertical and the crank's axis of rotation perpendicular to the direction in which the operator faced and located about 5 inches above the elbow level of the seated subject. A frictional torque, which varied slightly with rpm, was applied to the handwheel. The authors were primarily interested in the loads which the operators could overcome before losing control in their position-control tracking task. With one-handed tracking, 12 young subjects of average physique were studied to determine the speed at which loss of control occurred. The speed of rotation demanded by the task was increased slowly so that power was generated at a fairly constant level for between 10 and 30 seconds at the breakdown speed. The loss of control was manifest by lagging the target and then catching it by spasmodic efforts. In Figure 5-8, we have presented the average power output measured under these conditions. The power curve must, of course, return to zero as the tangential load approaches a magnitude which the operator cannot overcome. Actually, in the Hick and Clarke data, the approach to zero occurs because the speed of cranking at which tracking can be maintained approaches zero

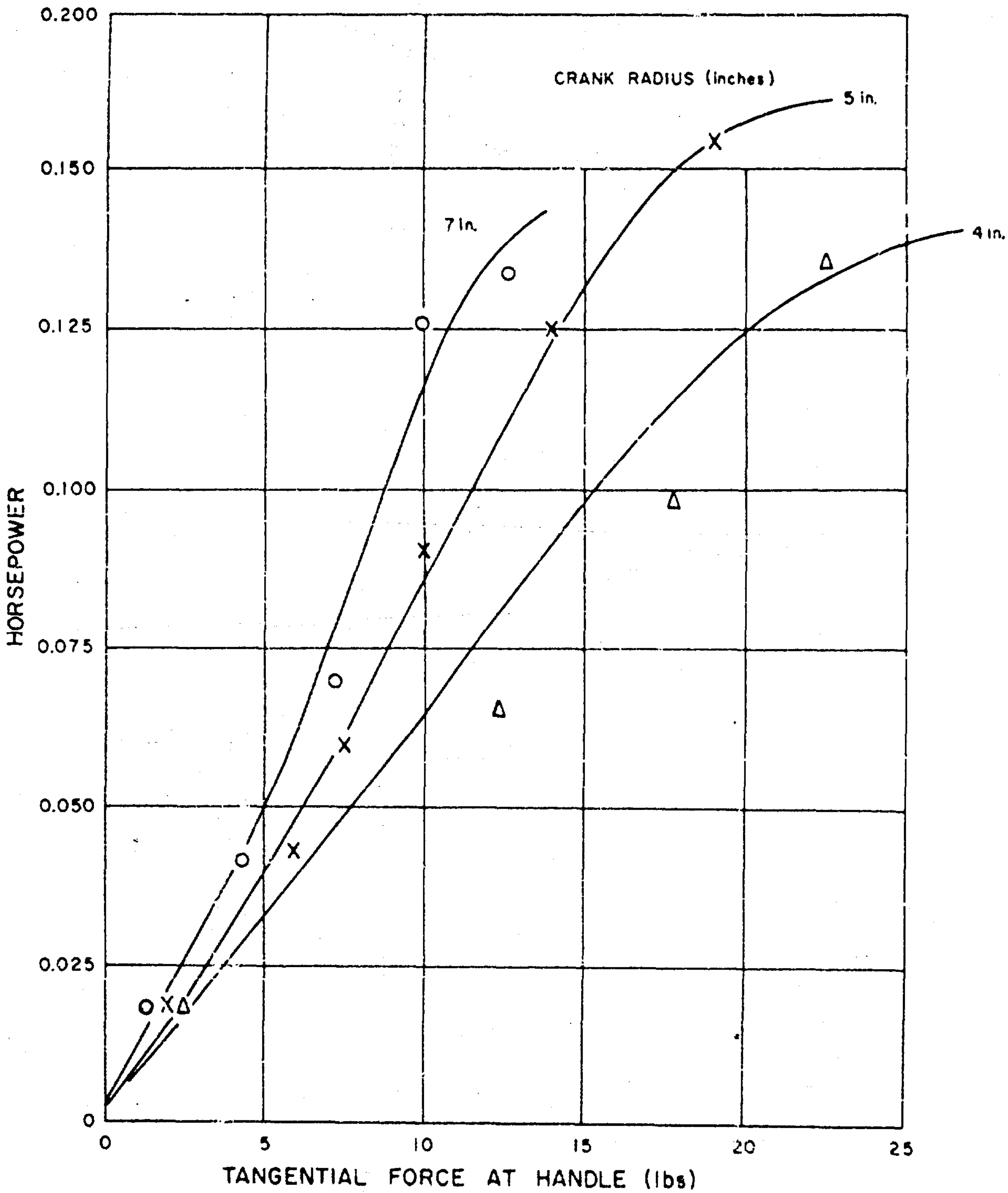


Figure 5-7. Self-Paced Cranking Power vs. Crank Force (from Katchmar, ref. 49)

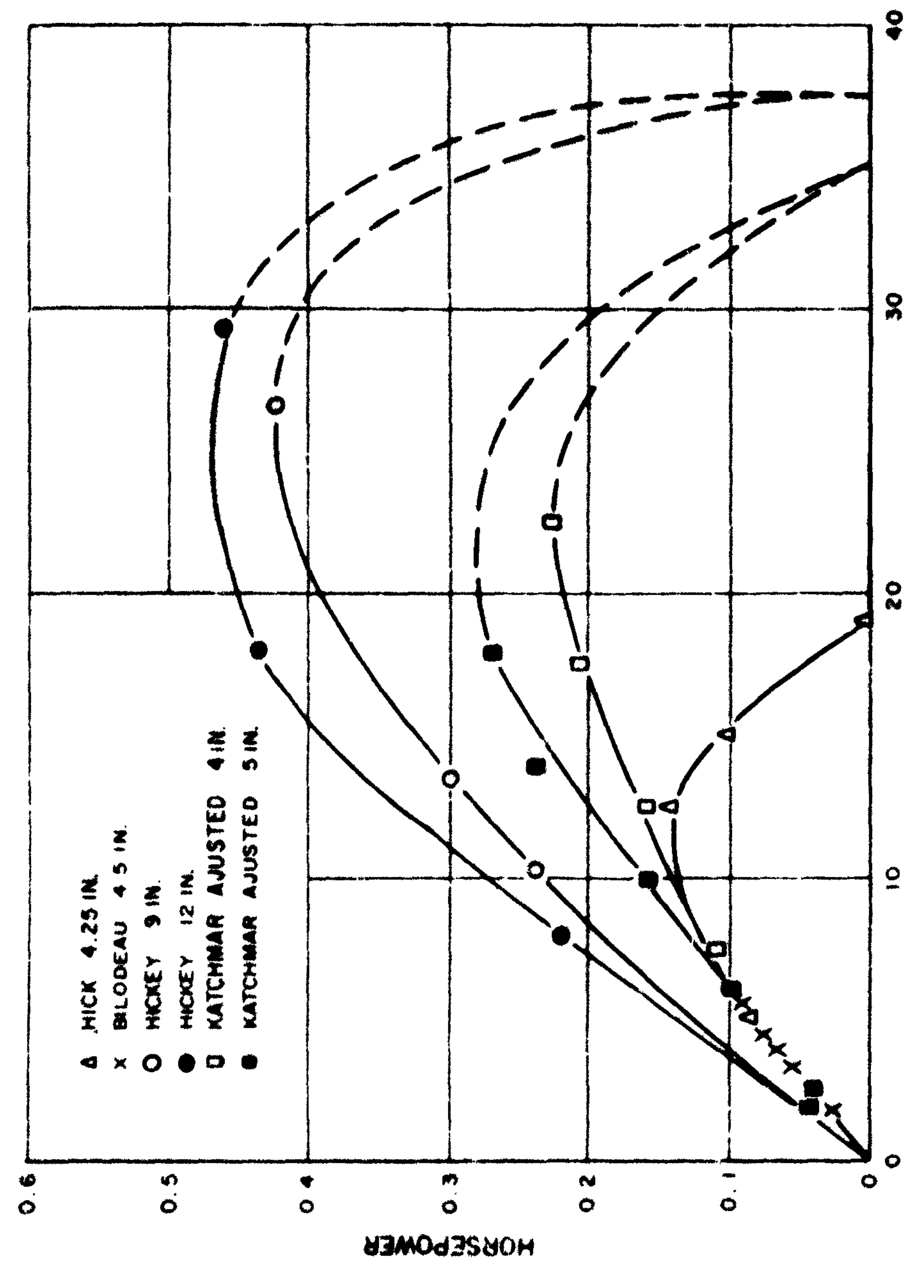


Figure 5-8. Average Power Generated by One-Handed Cranking (from Hick and Clarke, ref. 33)

with increasing load. The power was measured from the product of the average rotational speed and the frictional torque which was overcome. The speed at maximum power for these data was about 180 rpm. The approach to a constant slope of that portion of the power curve which extends to the origin represents the limitation on power generation due to the maximum speed at which the subject can rotate his crank. This maximum speed of rotation was about 260 rpm.

We have plotted the data from Figure 5-2a for the first 20 seconds of cranking on the same plot, and it will be seen that these data are consistent with the data of Hick and Clarke.

Using Equation (5-3) and some questionable assumptions, it is possible to adjust the self-paced data of Figure 5-7 to a form compatible with Figure 5-8. The Bilodeau and the Katchmar cranking tasks are similar in their physical characteristics. Thus, in Figure 5-2a the steady-state crank rotation for load No. 4 corresponds to the generation of about 0.04 hp against a tangential resistive force of 5.5 pounds. In the 5-inch radius self-paced task on Figure 5-7, we find 0.042 hp is generated against a 6-pound tangential load.

In order to convert the Figure 5-7 data to its sprint equivalent, let us assume that the foregoing point of contact between the tasks implies that m in Equation (5-1) is approximately equivalent for the Figure 5-6 and 5-2a data, and equal to 17 seconds; and that L is a function of loading only. Consequently, using Equation (5-3), we can compute the sprint horsepower for the first 20-second interval.

Since Figure 5-7 shows a unique horsepower to tangential loading function for each crank, the calculation of Equation (5-3) can make the Katchmar data compatible with Figure 5-8.

The 7-inch crank produced data inconsistent with that for the 4- and 5-inch cranks. The use of Equation (5-3) pushes the maximum power for 4- and 5-inch cranks out beyond the value determined under the tracking conditions; whereas, for the region of the curve where they are comparable,

the data from the three diverse sources are consistent in a satisfying manner.

It is of interest to note that Hick and Clarke, the Bilodeaux, and Katchmar each used a cranking axis at right angles to that used by the other two experimenters. Despite this, the data fall on the same line in Figure 5-8. This would indicate that for small cranks, position effects are of minor importance. Findings by Reed(53) for cranks of radii between 0.8 and 2.8 inches, and very light loads, indicate that whether the cranking be clockwise or counterclockwise is also relatively unimportant for small cranks.

Power generation by cranking through larger radii was investigated by A. E. Hickey of the Electric Boat Company(34). His apparatus consisted of a crank mounted to rotate in a verticle plane at the edge of a sturdy table. The crank axis of rotation was 36 inches from the floor, and the crank radius of rotation could be set at either 9 or 12 inches. The 9-inch crank was turned counterclockwise and the 12-inch crank was turned clockwise. A prony brake supplied the resistive torque. The subjects were naval ratings who stood in front of the crank and rotated it as fast as possible. The subjects usually each made two trials, one for a total of 20 turns and a second trial for a total of 30 turns, for each value of the torque loading on the crank. In only one case was the time duration for the 30-turn trial as high as 40 seconds. In all the other cases the duration was close to 20 seconds for the 30-turn session. Since the power generated in the 20- and 30-turn trials was about equal, they were averaged together. Between four and ten trials were averaged for each data point. Some of the subjects were used for different crank loadings, but only the sturdiest subjects were capable of operating under the heaviest loads. Figure 5-8 presents Hickey's data up to the point where the subjects spontaneously shifted from one- to two-handed cranking. On the assumption that this point represented maximum effort power generation for the crank used, Hickey's data was extrapolated to intercept zero at an estimated force loading. Using the Müller and Müller data for two-handed handwheel operation(47), assuming the maximum force for

one hand is half that for two hands, one would get an isometric force of about 90 pounds for an instantaneous effort. Since we are interested in forces maintained for about 20 seconds, the x intercept of Figure 5-8 is reasonable. The speed of rotation for the 9-inch crank maximum power production was about 110 rpm and about 85 rpm for the 12-inch crank. The maximum limiting speeds, as measured from the constant slope portion of the curves extending to the origin, were about 170 rpm for the 9-inch radius crank, and about 150 rpm for the 12-inch radius crank.

On Figure 5-9, we have plotted the Hickey data after the subjects shifted to two-handed operation of the single-handle crank. The maximum for power output for the 9-inch crank occurs at about 85 rpm, and the maximum is about 80 rpm for the 12-inch crank. The limiting two-handed speed of rotation was 30 rpm for the 9-inch crank, and 100 rpm for the 12-inch crank.

Hick and Clarke also studied two-handed winding, using 6 subjects with essentially the same apparatus as was previously discussed, save for the presence of an additional crank at a 9-inch separation from the first crank. The twisting moment imparted by this reciprocal winding was not a problem for the operator. On Figure 5-9, we have plotted these data for 4.25-inch radii of cranking. Maximum power occurs at about 140 rpm, and the limiting speed of rotation is about 210 rpm.

The development of useful power by impulsive outputs of energy has been studied for both conditions in which the muscular involvement was large as well as for more restricted muscular involvement as might be involved in a manipulative production line task. The generation of power over such miniscule intervals as occur in kicking a football, punching a bag, swinging an axe, will not be considered here. Such episodes, however interesting, are more naturally characterized by the impulse momentum, i.e., $\int_0^T F dt = I$, rather than by their energy or power characteristics.

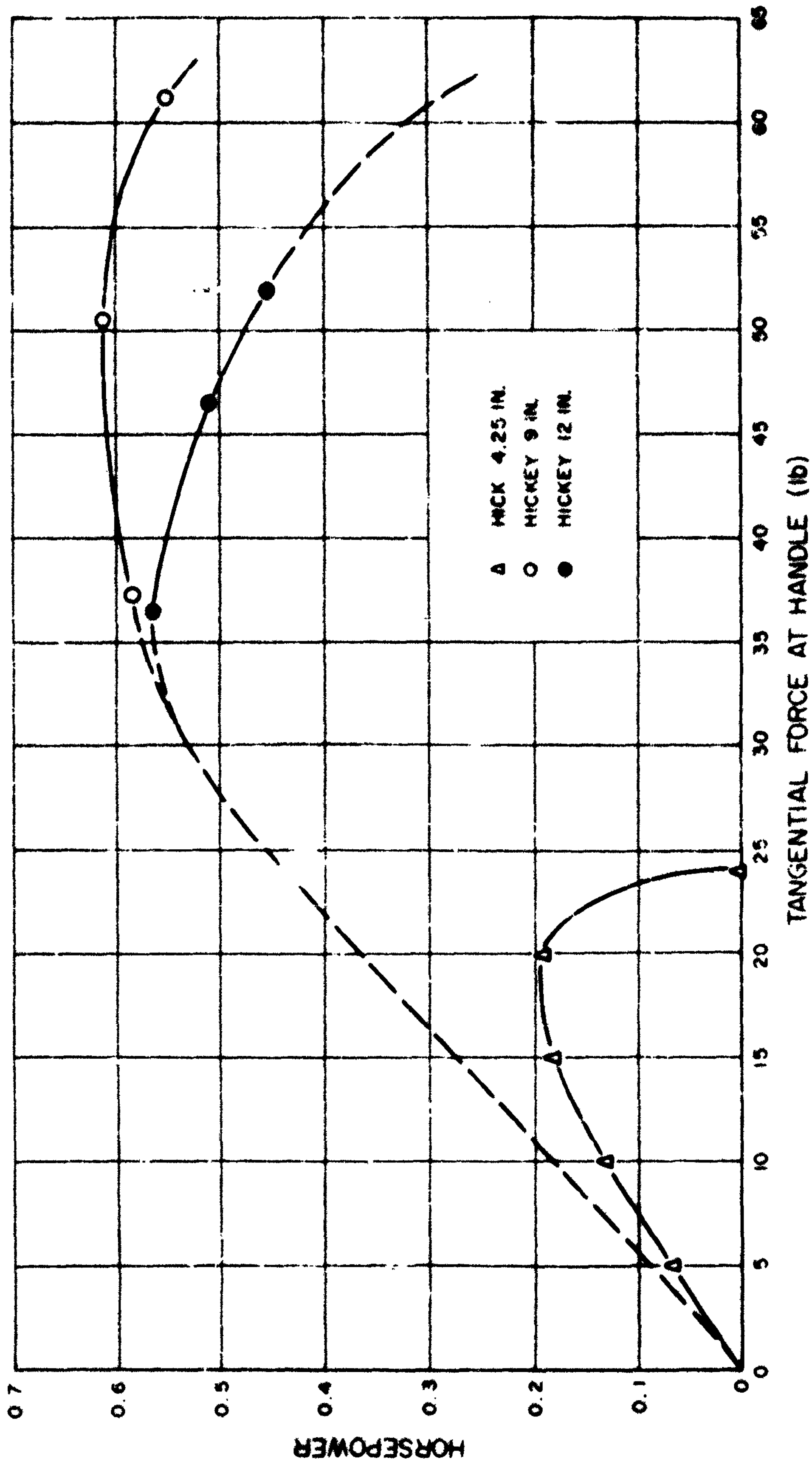


Figure 5-9. Average Power Generated by Two-Handed Cranking (from Hickey, ref. 38)

In order to obtain information on the types of motions permitting the greatest velocity and horsepower for the design of tools, jigs, work methods, etc., Koepke and Whitson conducted an experiment(41) on six men whose ages varied from 21 to 30. Six different weights, 6, 9, 12, 15, 18, and 21 pounds, respectively, were accelerated by the subjects using six types of right-hand motions. The motions were:

(1) Long forehand sweep from right to left with the arm extended.
(2) Long backhand sweep from left to right with the arm extended.
(3) Short forehand sweep from right to left with the forearm only, the elbow being held at the side.
(4) Short backhand sweep from left to right with the forearm only, the elbow being held at the side.
(5) Forward thrust of the right arm from a position at the right side of the body.
(6) Pull of the right arm toward the body from an extended position of the front of the body.

Determinations were made of the time history of the velocities attained and the useful power expended by each of the subjects under the foregoing conditions. These determinations were made with the subjects in a sitting position. No definite termination points were specified in the motions since the effect on the subject of anticipating a stop was considered undesirable. The weights were taken in order of their magnitude, starting with the heaviest. The six prescribed manipulations of the weights were made in the order previously presented. All of the motions with one weight were completed before going on to the next. A brief pause was taken between motions.

Time intervals of 0.01 seconds were used to determine accelerations and velocities of the weight, and maximum instantaneous horsepower magnitudes were computed for the duration. The general findings of interest to us are as follows:

(1) The variability between subjects was considerable. (See Figure 5-10)
(2) The maximum instantaneous horsepower is fairly strongly dependent on the type of motion but is not very sensitive to the weight moved in the range studied. (See Figure 5-11)

(3) Averaging overall subjects and motions, the maximum instantaneous horsepower is independent of the weight moved over the range studied. (See Figure 5-12.)

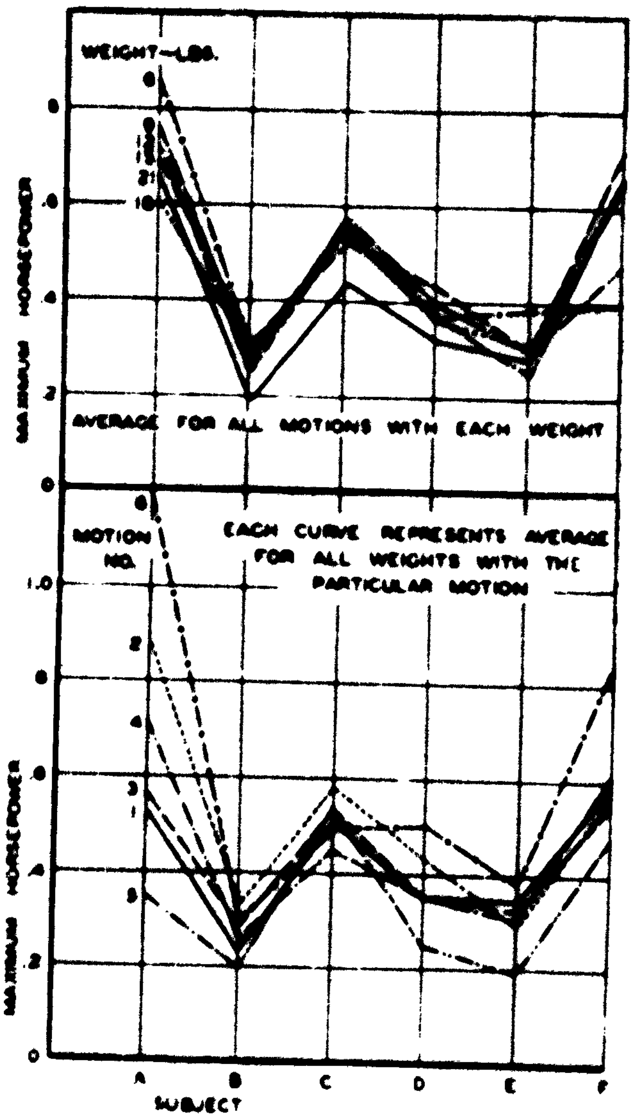


Figure 5-10. Variation of Maximum Horse Power Between Subjects (from Koepke and Whitson, ref. 41)

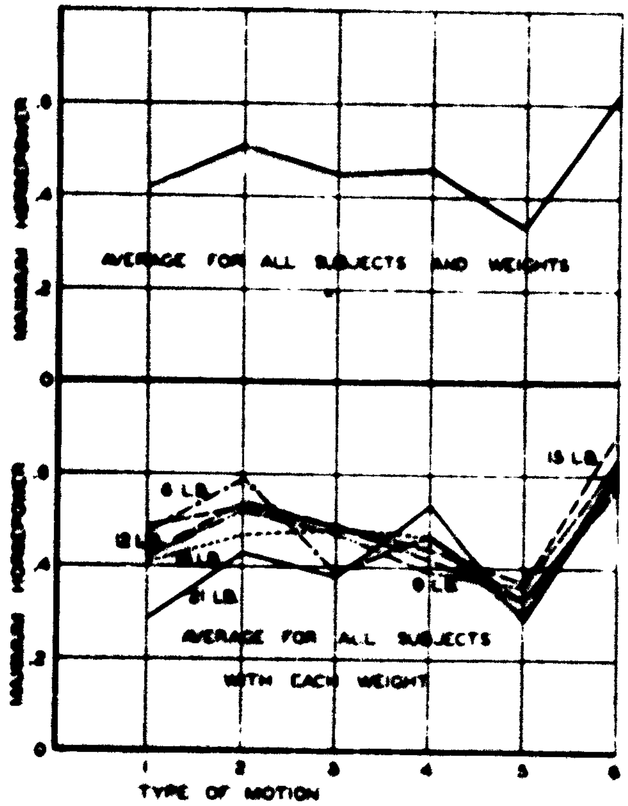


Figure 5-11. Relationship Between Type of Motion and Maximum Horsepower (from Koepke and Whitson, ref. 41)

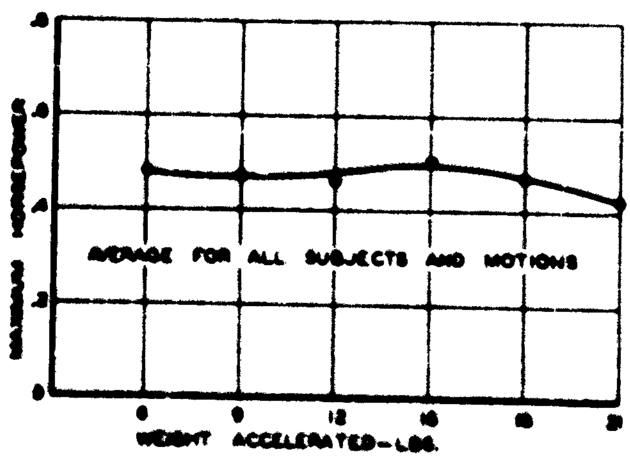


Figure 5-12. Relationship Between Weight Accelerated and Maximum Horsepower Attained (from Koepke and Whitson, ref. 41)

It is of interest to note that the maximum instantaneous hp with motion No. 6, which was grossly similar to the elbow motion in the Lupton studies discussed in Section III, has a comparable maximum value (see Equation 3-8). The fact that this maximum is much greater than Wilkie(59) predicted from Figure 3-3 might indicate that Wilkie's refined apparatus, by limiting the musculature involved, may have resulted in a decrease in the effort that the subject could produce as compared with Lupton's(45) somewhat less rigid experiment, and as further compared with the free movement of Koepke and Whitson's motion No. 6.

In addition, Koepke and Whitson's data are measured over a 0.01-second duration and are thus closer to instantaneous power.

The Lupton power maximum was computed on the assumption that the power was maintained over the minimum contraction time of 0.26 seconds. This time over which the maximum velocity is relatively constant is a function of the load, and this may help to explain the difference between the Lupton and the Wilkie maximum power values. Furthermore, as previously noted, the manipulations to which the Lupton data were subjected would easily permit errors to creep in.

(4) The time to reach the maximum instantaneous horsepower can be seen as a function of the weight and of the type of motion. (See Figure 5-13.)

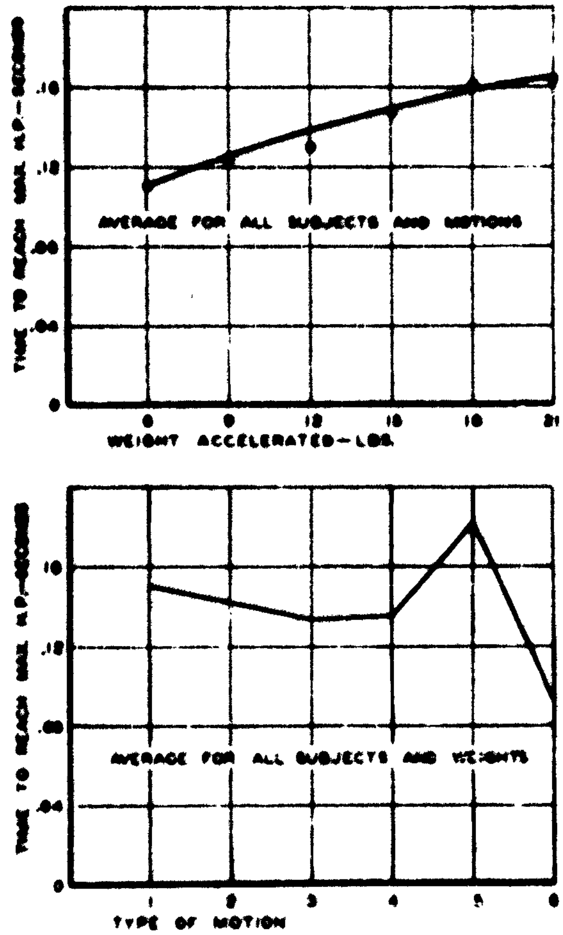


Figure 5-13. Upper Curve: Relationship Between Weight Accelerated and Time to Reach Maximum Power; Lower Curve: Relationship Between Type of Motion and Time to Reach Maximum Horsepower (from Koepke and Whitson, ref. 41)

The foregoing findings together with the data on the velocities achieved in each motion have been applied to production problems. Many other applications are apparent.

The generation of power by a pedal-operated device is probably the technique with which we are most familiar. This method of power generation has been commonly used in bicycling for ground locomotion and uncommonly used in muscle-powered flight for aerial locomotion! It is, therefore, rather surprising that so much of the data on long duration as well as short duration efforts in bicycling type tasks are of a testimonial rather than of a well-documented scientific nature. Much of the difficulty in obtaining comparable data results from the multitude of variables which are associated with bicycling as was mentioned in Section IV. In addition, in many cases, it is hard to determine whether the task performed was maximum effort or self-paced. Since we cannot, in general, specify crank size, leg motion, average rpm of pedal, etc. for the pedalling data, they will be presented without detailed comment.

During the years of the twenties and thirties, about four and one-half centuries after Leonardo da Vinci designed an ornithopter, interest in muscle-powered flight flourished in Germany, France, Italy, and, to some extent, in the Soviet Union. Although both ornithopters and propeller-driven vehicles were studied, the propeller-driven vehicle appeared to be most promising. The essential function of muscle power was to prolong the flight of highly efficient gliders by aiding the low sinking speed characterizing such aircraft(51,57).

As a consequence of this interest in muscle-powered flight there exists an arcane German literature on human power generation by means of pedalling as well as by means of handwheel rotation. One of the theories about human power generation which was current among the "muskelflugers" was one attributed to a Dr. Brustmann who apparently held that the time integral of the power curve had an upper limit of 80,000 kg-m. The precise origin of this figure as well as the functions describing the time history of this

limiting power-generation are unknown to this writer, although it appears to arise from the asymptote to the envelope of maximum power-generation curves under all types of exertions(30,43). In Figure 5-14, data presented by Haessler(30), without detailed comment, are shown. The upper curve was generated by a well-known, highly-trained racing cyclist. The lower curve was generated by a strong, but not especially well-trained cyclist. One of the Benedict and Cathcart(3) subjects, a trained athlete (M.A.M.) pedalled the bicycle ergometer at a sustained effort for 4.5 hours. His data point is included on the upper curve. The odd point at 60 minutes was the average of the highest power outputs produced by two subjects of Garry and Wishart(29) pedalling for an hour under instructions that they maintain a given speed without showing fatigue. Whether this speed could have been maintained for a larger interval is not clear from the reference cited.

The influence of test pauses on power generated was studied by Crowden(16) with a healthy young bicyclist as his subject. A comparison was made between bicycling continuously for six minutes at the sub-maximum effort, but optimally efficient cranking rate of 33 rpm, and bicycling for 20 seconds at 101 rpm - resting for 40 seconds. Although pedalling in a maximum-effort spurts was less efficient, the average horsepower was higher than for the continuous case. If we disregard the last 40 seconds of the continuous run so as to avoid averaging the last 40 seconds of zero effort in the intermittent run, we obtain the following averages for power generated over 320 seconds:

Continuous effort = 0.13 hp.
Intermittent effort = 0.16 hp.

The continuous effort data is the average of two 6-minute runs. If we compare over 6-minute runs we have:

Continuous effort = 0.13 hp.
Intermittent effort = 0.14 hp.

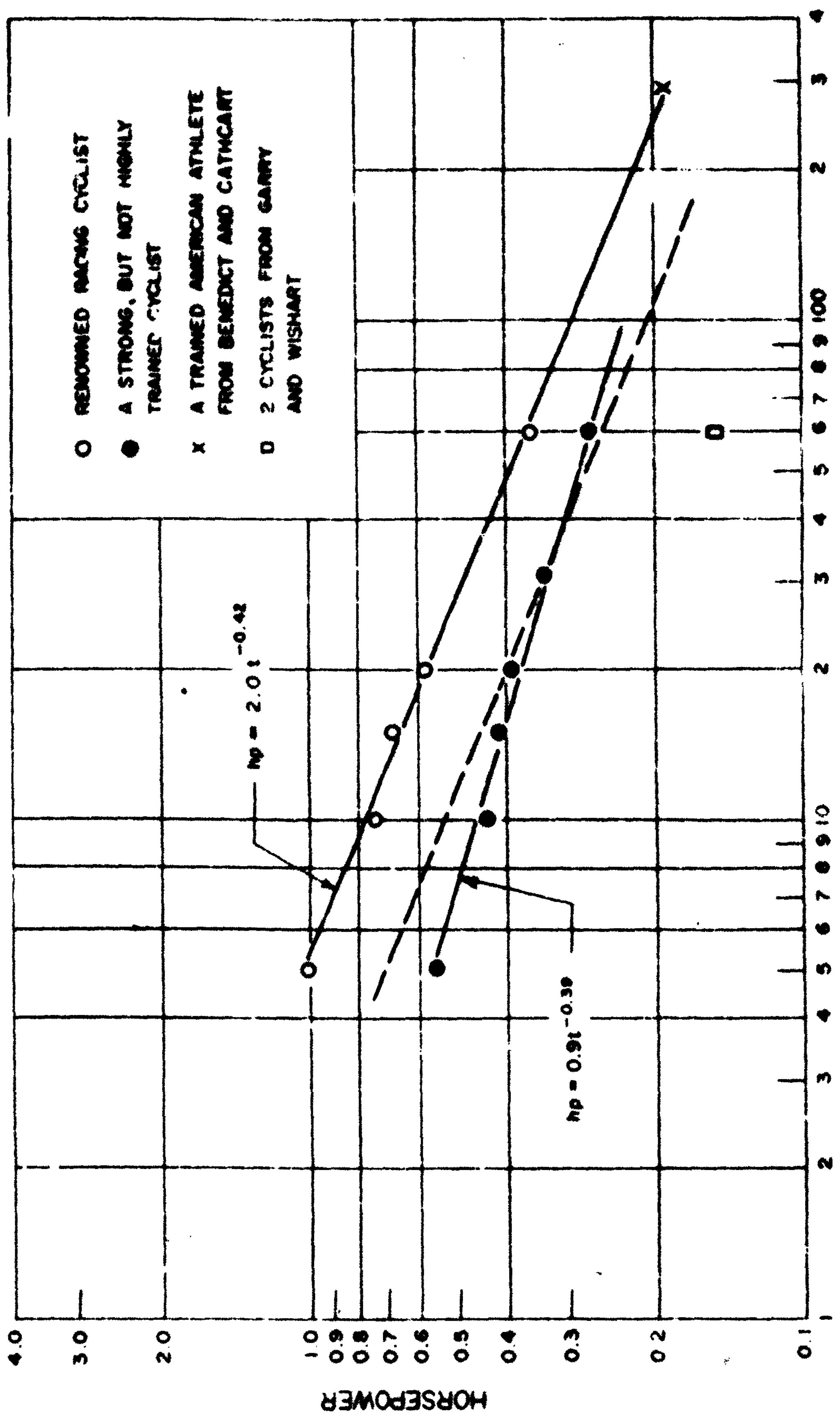


Figure 5-14. Long Duration Pedalling Power (from Bussler, ref. 38)

The work per leg movement was kept constant for both rates of cranking. Thus, although the slower rate of cranking is more efficient metabolically, the maximum effort cranking is more effective, over approximately 6-minute intervals, in producing power. It is to be expected, however, that over long periods of effort the average horsepower generated at a maximally efficient speed will exceed the average generated by combinations of other speeds and rest pauses.

In a submaximal effort bicycling experiment, Müller(46) demonstrated that for a given total rest pause duration many small pauses were more desirable than fewer long pauses. His primitive experimental design precludes any quantitative use of these data.

Bujas and Petz(12) obtained results confirming Müller's findings in that the average power output is greater for shorter pauses in a maximum effort static (i.e., purely internal work) task than it was for longer pauses. Recovery from fatigue proceeded at a higher rate when the subject worked against a heavy load than when he worked against a light load.

In Figure 5-15 we have replotted certain short time duration power-generation data obtained by Ursinus(58) by having the subject pedal, crank a wheel, and perform both acts simultaneously. These data are probably not representative, since the subject was very highly trained. It will be noted that the Crowden data fall considerably below the comparable Ursinus data points. The pedalling rate in the Ursinus data was 108 rpm. The horsepower versus time function has no significance below t = 10 seconds. The curve fits yield the following approximations:

Mode of Power Generation		
arms	$hp = 1.5t^{-.40}$	
legs	$hp = 2.8t^{-.40}$	$10 \leq t \leq 300$ (sec)
arms & legs	$hp = 4.4t^{-.40}$	

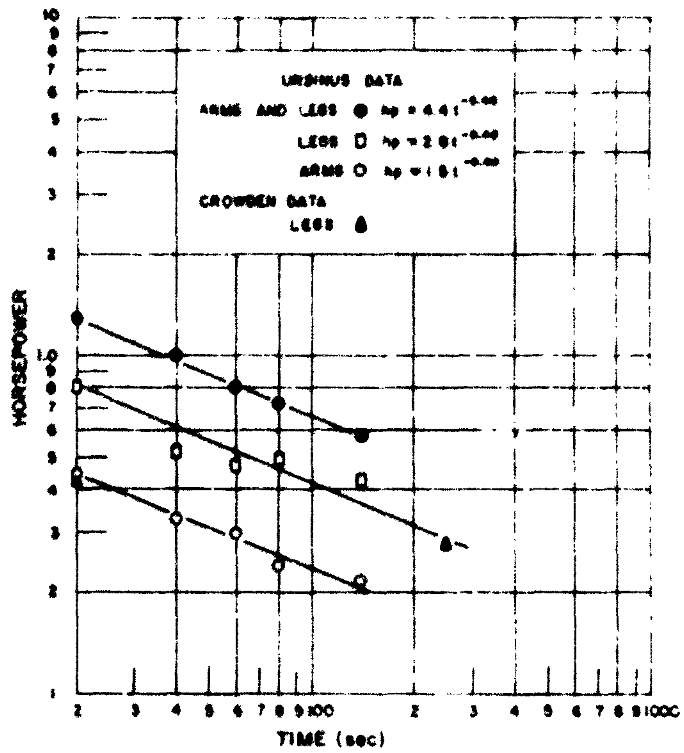


Figure 5-15. Maximum Effort Cranking and Pedalling Power (from Ursinus, ref. 58)

Figures 5-14 and 5-15 fitted as they are by the Equation (5-2) form, provide a temptation to extend the notion of the self-paced task to the skilled athlete's power output. It may be that a skilled athlete (all of the subjects in Figures 5-14 and 5-15 were skilled to some extent) has learned how to pace his output in such a fashion that he doesn't emit a wasteful burst of power early in the course of his activity and thus incur an oxygen debt which will rapidly increase and fatigue him. Instead he may work at such a rate that his oxygen debt rises rather slowly until he must finally stop his efforts. This may constitute a general self-pacing behavior. This argument is, of course, far from compelling. It is of interest to note the consistency of the long time duration power in Figures 5-14 and 5-15 with the maximum magnitude of 0.5 to 0.4 hp mentioned in Section IV.

Bossi and Bonomi(56) in Italy present a curve showing the generation of instantaneous power, as a function of speed of rotation, by Emile Casco, an extremely powerful cyclist, as well as by Hofmann; the most powerful pilot previously tested at the Muskelflug Institute. These data are shown on Figure 5-16. Comparing Figure 5-16 with Figure 5-15 it would appear that Hofmann was the pilot from whom Ursinus obtained the data in Figure 5-15, and that the instantaneous power was measured over a 10-second interval.

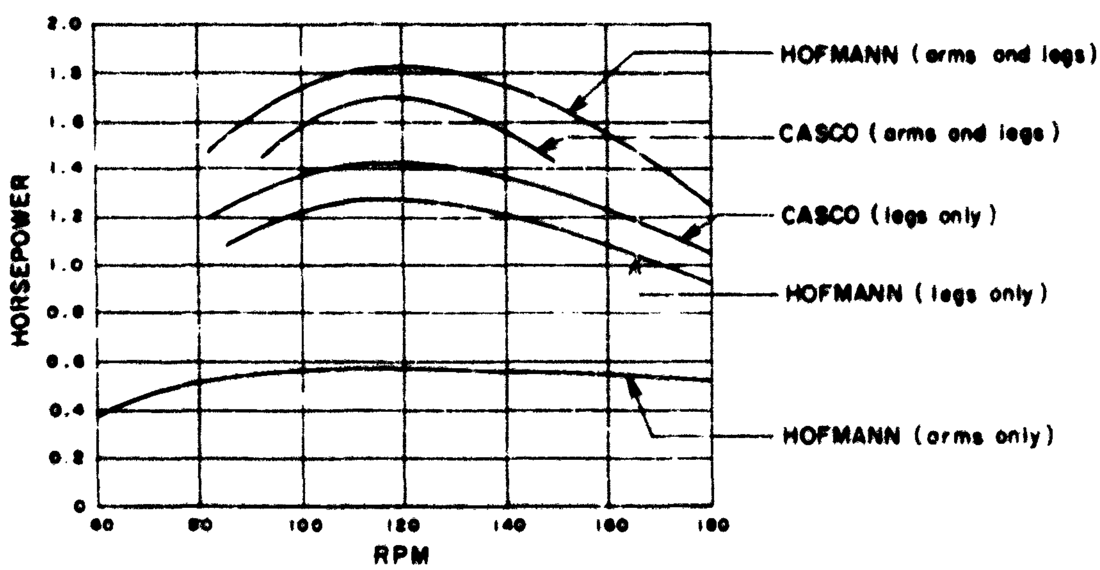


Figure 5-16. Generation of Instantaneous Power as a Function of Speed of Rotation (obtained by Bossi and Bonomi, ref. 56)

There exists very little convincing data about the gross involvement of the human musculature over long and short durations in power generation by means of other than simple, conventional mechanical systems.

Rowing is included in this category, since the magnitudes of power produced are similar to those produced in running and climbing. Henderson

and Haggard(32) present data on the Yale University Crew which won the 1924 Olympics. These data indicate that 0.57 hp were generated per man over a 4.8 minute duration. The data, however, were obtained by towing a loaded shell at the end of an ice scale at the same speed attained by the crew. Wasted effort, from the viewpoint of propelling the shell, was arbitrarily assumed to be 25%, and this approximation diminishes the value of this frequently quoted horsepower rating for a championship rowing crew.

Lippisch(43) has presented data which imply that a sprinter generates about 7 hp; a racing oarsman about 2 hp over ten minutes, and continuous power can be generated at about 0.25 hp over a period of one hour. These data, however, are suspect since the measures for the sprinter and oarsman are presumably useful power plus wasted power: whereas, the 0.25 hp is presumably useful power only. (See Figure 5-__.)

To complete this summary of power generated by gross body musculature, we have the following data on climbing. These data consider that work done should be a change in potential energy only; hence they present power values which are much less than the Hill or Fenn data for sprinting.

Table 5-1 (from Blix[10])

Work	Duration	Average hp
Mountain climbing, moderate	many hours	0.11
Mountain climbing, severe	1-2 hours	0.16-0.22
Steep, 100 meters	3 3/4 minutes	0.44
Climbing a treadmill	30 seconds	0.52-0.78
Running upstairs with 10 kgm load	15 seconds	0.81
Running stairs, no load	30 seconds	0.94
Running stairs, no load	4 seconds	1.24-1.30

Other data related to climbing are for the Ben Nevis climb of 4300 feet in 1 hour, 8 minutes and 19 seconds, or an average hp of 0.27(55).

There are several remaining aspects of the problem of characterizing man-generated power by means of mechanical devices. Motivation, individual variability, postural effects, and performance with respect to criteria other than power production, are a group of miscellaneous aspects

which are centered more or less on the general problem of individual differences.

Motivation is the easiest of the foregoing factors to discuss. It is clearly important, but no relevant quantitative data were found.

Some quantitative measure of individual variability in a cranking task may be obtained from the Bilodeau data(8). In Table 5-2 we present the standard deviation to mean ratio, σ/M, for each of the twelve 10-second intervals which comprised the two minutes of cranking for which M was plotted on Figures 5-2b and 5-3. The number of subjects for each determination was 40.

Table 5-2

RATIO OF STANDARD DEVIATION TO MEAN

Cranking Schedule		1	2	3	4	5	6	7	8	9	10	11	12	average σ/M
Sprint 3-10	(Fig. 5-2b)	.08	.07	.08	.09	.11	.11	.11	.12	.11	.11	.11	.12	.10
30-10		.08	.09	.09	.10	.10	.10	.10	.11	.12	.13	.10	.11	.10
3-3		.15	.12	.11	.09	.09	.09	.11	.09	.09	.10	.10	.09	.10
Self-paced 3-5		.15	.14	.13	.12	.11	.12	.12	.11	.11	.12	.11	.12	.12
30-3	(Fig. 5-3)	.13	.12	.10	.10	.10	.09	.09	.09	.09	.09	.08	.08	.10
30-5		.14	.12	.12	.11	.11	.11	.11	.12	.12	.12	.13	.12	.12

Table 5-2 demonstrates a difference in trend between the σ/M ratio for sprint power output as compared with the σ/M ratio for self-paced power output. For simplicity one can assume that this variation with time is negligible, and further assume that a fairly constant σ/M ratio characterizes other human power generation tasks. These assumptions enable us to convert data obtained from exceptionally proficient athletes (see Figure 5-14) to data which might characterize the mean of the appropriate population of athletes.

Consider the renowned cyclist in Figure 5-14 to be three standard deviations above the mean of a population of good cyclists. From Table 5-2, let σ/M for this population be 0.12. Therefore, if M_r is the measured performance of the renowned cyclist, and the population mean is M,

$$M_r = M + 3\sigma = M + 3\ (0.12M)$$

$$M = M_r/1.36 \approx .7\ M_r \qquad (5\text{-}4)$$

On Figure 5-14, the dotted line parallel to the renowned cyclist's curve was drawn according to Equation (5-4). The ratio, σ/M, is clearly not a constant for this situation, and it actually appears as if it has the general characteristic shown in Table 5-2 for self-pacing cranking tasks. Similar cavalier treatment of the data could be applied to Figures 5-15 and 5-16 in order to obtain better estimates of the performance of average members of the population.

Postural effects in the generation of power relate to such matters as anthropometric measurements, and the location of controls, appropriate harnesses and backrests(17,18,57), and the definition of functional areas for working such that geometric position has associated with it a weighting which denotes the operators ability to generate a force or to do work(54).

As a general rule, postural or positioned effects are unimportant when relatively small amounts of power are generated, as with cranks of radii between 4 and 5 inches. The comparisons on Figure 5-8 bear out this point since the cranks compared were rotated about all three orthogonal axes. Although much work has been done with regard to the optimal positioning of larger cranks and wheels, the maxima about reasonable positions are fairly flat. To be sure, large work decrements have been demonstrated for absurd positions of cranks, but common sense is about as good a guide as the data available. It appears to be true that the position of maximum force and endurance is also that position for which submaximal forces may be applied with least fatigue. The position of maximum force, however, is

not generally equal to that position from which most precise control may be exerted or that position which is most comfortable(17,42).

Backrests, harnesses, nonslip flooring - all contribute to greater power output by so positioning the operator that he may assume a favorable mechanical relationship with his machine without having to dissipate some of his own energy in maintaining this position. The particular design of such devices cannot be discussed independently of the particular machine being operated.

Finally, a remark about performance with regard to criteria other than power production is in order. In the Hick and Clarke study of power generation in a tracking task(33), both variability and absolute error increased markedly with increased power production. Generally speaking, the well-known increase in variability with increasing fatigue will deteriorate performance, measured by criteria other than power generation, in a power generating task.

SECTION VI

CONCLUSIONS AND RECOMMENDATIONS

There have been two different goals in the foregoing sections:

(1) An effort was made to organize with respect to the orientation of Section II, and to compare, such data as were available on human power production. The paucity of these data made some rather questionable extrapolations necessary in an effort to fill in the gaps.

(2) An effort was made to indicate a scheme which could be applicable to the design of man-machine systems for the most efficient power transfer from the man to the machine.

Although one might readily suggest that the many gaps in the existing data should be filled by a series of laborious, painstaking (and probably uninspired) experiments, this is not the opinion of this writer. The second of the goals of this report provides much greater promise for a useful end product. In the foregoing sections the work conducted by Hill's school, culminating in Wilkie's data, has demonstrated that measurements and theoretical considerations related to the microstructure of the human musculature, i.e., the individual muscle fibers, can be used to predict the performance of the gross anatomy of the body. Evidence that the individual muscle possesses a measurable compliance was introduced(59,61). These facts, with their implications that the impedance-matching concepts of Section II are realizable, are the basis of our recommendations for future work.

A program to implement Section II should be carried out so that the dynamics of human muscular activity can be studied in this framework. The goal of this program should be the measurement of such dynamic parameters of human behavior as compliances and masses in simple power-production tasks. These measurements should be made in the intact, living organism, unlike certain classic data on this subject(11). A start has been

made in this direction by the various attempts to characterise the masses, and the moments of inertias of various limbs about selected axes(11,26,59).

This program should proceed by attempting to design, construct, and then to test simple mechanical devices designed from optimal power transfer consideration. The positive results of a successful investigation will yield both obvious practical advantages, as well as the more fundamental benefits which will be derived from knowing more about the dynamics of human mechanical activity.

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BIBLIOGRAPHY

1. Abbott, B.C., Bigland, B., and Ritchie, J.M., "The Physiological Cost of Negative Work", Journ. of Physiol., 117, 1952, pp. 380-290

2. Asmussen, Erling, Experiments on Positive and Negative Work, Ergonomics Research Society, Symposium on Fatigue, Floyd, W.F. and Welford, A.T. (ed.) H.K. Lewis and Co., Ltd., 1953, pp. 77-82

3. Benedict, F.G. and Cathcart, E.P., Muscular Work, Carnegie Institute of Washington Publication, 187, 1913

4. Best, C.H. and Partridge, R.C., "The Equation of Motion of a Runner Exerting a Maximal Effort", Proc. Roy. Soc. of London, Series B, 103, 1928, p. 218

5. Bilodeau, E.A., "Massing and Spacing Phenomena as Functions of Prolonged and Extended Practice", Journ. Exp. Psychol., 44, 1952, pp. 108-113

6. Bilodeau, E.A., "Decrements and Recovery from Decrements in a Single Work Task with Variation in Force Requirements at Different Stages of Practice", Journ. Exp. Psychol., 44, 1952, pp. 96-100

7. Bilodeau, E.A., "Rate Recovery in a Repetitive Motor Task as a Function of Successive Rest Periods", Journ. Exp. Psychol., 48, 1954, pp. 197-203

8. Bilodeau, I. McD., "Performance of an Effortful Task with Variation in Duration of Prior Practice and Anticipated Duration of Present Practice", Journ. Exp. Psychol., 46, 1953, pp. 146-153

9. Bilodeau, I. McD. and Bilodeau, E.A., "Some Effects of Work Loading in a Repetitive Motor Task", Journ. Exp. Psychol., 48, 1954, pp. 455-467

10. Blix, Scand. Archiv., F. Physiol., 15, 1904, p. 122 (as reported in Benedict, F.G. and Cathcart, E.P., Muscular Work, Carnegie Institute Publication, 187, 1913)

11. Braune, W. and Fischer, O., Abh. d. math. phys. klin. d. Sachs Akad. d. Wiss., 18, 1892, p. 409 as reported in Fenn(25) and Wilkie(59)

12. Bujas, Z. and Petz, B., "Endurance and Recovery Performance of Static Work", Arhiv za higijenu rada, 4, 1953. pp. 349-363

13. Campbell, J.M.H., Douglas, C.S., and Hobson, F.G., "The Respiratory Exchange of Man During and After Muscular Exercise", Phil. Trans. Roy. Soc., Series B, 210, 1921, pp. 1-47

14. Clark, D.K., A Manual of Rules, Tables and Data for Mechanical Engineers, 6th ed., 1893, pp. 718-720

15. Coulomb, Memoires de l'Institut National des Sciences et Artes: Science, Math. et Phys., T II, commented on by Poncelet, Mechanique Industrielle, Paris, 1841, P. 237, and presented in Smith, J. Jervis, edited by Boys, C.V., Dynamometers, D. Van Nostrand, N.Y. 1915, pp. 10-11

16. Crowden, G.P., "The Effect of Duration of Work on Efficiency of Muscular Work in Man", Journal of Physiol., 80, 1935, pp. 394-408

17. Darcus, H.D., "The Range and Strength of Joint Movements", Human Factors and Equipment Design, Floyd, W.F. and Welford, A.T. (ed) London, 1954, pp. 37-46

18. Dempster, W.T., "The Anthropometry of Body Action", Annals of the New York Academy of Sciences, 63, 1955, pp. 559-585

19. Dern, R.J., Levene, J.M., and Blair, H.A., "Forces Exerted at Different Velocities in Human Arm Movements", Amer. Journ. Physiol., 151, 1947

20. Dickinson, S., "The Dynamics of Bicycle Pedalling", Proc. Roy. Soc. of London, Series B, 103, 1928, pp. 225-233

21. Dickinson, S., "The Efficiency of Bicycle Pedalling as Affected by Speed and Load", Journ. of Physiol., 67, 1929, pp. 242-255

22. Dill, D.P., "The Economy of Muscular Exercise", Physiological Reviews, 16, 1936, pp. 263-289

23. Fenn, W.O., "Frictional and Kinetic Factors in the Work of Sprint Running", Amer. Journ. of Physiol., 92, 1930, pp. 583-618

24. Fenn, W.O., "Work Against Gravity and Work Due to Velocity Changes in Running", Amer. Journ. of Physiol., 93, 1931, pp. 433-462

25. Fenn, W.O., and Marsh, B.S., "Muscular Force at Different Speeds of Shortening", Journ. of Physiol., 85, 1935, pp. 277-297

26. Fenn, W.O., "The Mechanics of Muscular Contraction in Man", Journ. of Applied Physics, 9, 1938, pp. 165-177

27. Fenn, W.O. "Contractility", a chapter in Physical Chemistry of Cells and Tissues, by Hober, R., York, Pa., Blakiston, 1945, pp. 445-447

28. Furusawa, K., Hill, A.V., and Parkinson, J.L., "The Dynamics of Sprint Running", Proc. Roy. Soc. of London, Series B, 102, 1927, p. 29

29. Garry, R.C. and Wishart, G.M., "Efficiency of Bicycle Pedalling", Journ. of Physiol, 72, 1931, pp. 425-437

30. Haessler, H., "Über die Durchführbarkeit des Muskelkraftfluges", Flugsport, 26, 1934, pp. 2-6

31. Haldane, J.S., and Henderson, Y., "The Rate of Work Done with an Egyptian Shadouf", Nature, 118, 1926, pp. 308-309

32. Henderson, Y. and Haggard, H.W., "The Maximum of Human Power and Its Fuel", Amer. Journ. of Physiol., 72, 1925, p. 264

33. Hick, W.E. and Clarke, P., The Effect of Heavy Loads on Handwheel Tracking, Medical Research Council Applied Psychology Research Unit, Cambridgo, England, R.N.P. 313, G.S. 97, Aug. 1946

34. Hickey, A.E. Jr., Personal Communication, Electric Boat Co., Groton, Conn., March 7, 1957

35. Hill, A.V., "The Maximum Work and Mechanical Efficiency of Human Muscles and Their Most Economical Speed", Journ. of Physiol., 56

36. Hill, A.V., "The Laws of Muscular Motion", Proc. Roy. Soc. of London, Series B, 100, 1926, pp. 87-108

37. Hill, A.V., "The Heat of Shortening and the Dynamic Constants of Muscle", Proc. Roy. Soc. of London, Series B, 126, 1938, pp. 136-195

38. Hill, A.V., "A Discussion on Muscular Contraction and Relaxation: Their Physical and Chemical Basis", Proc. Roy. Soc. of London, Series B, 137, 1950, pp. 40-87

39. Hill, A.V., "The Mechanics of Voluntary Muscle", The Lancet, 1951, pp. 947-951

40. Katchmar, L.T., Physical Force Problems: 1. Hand Crank Performance for Various Crank Radii and Torque Load Combinations, Tech Memo No. 3-57, Human Engineering Laboratory, APG, Md.

41. Koepke, C.A. and Whitson, L.S., "Power and Velocity Developed in Manual Work", Mechanical Engineering, May 1940, pp. 383-389

42. LeGros Clark, W.E. "The Anatomy of Work", Human Factors and Equipment Design, Floyd, W.F. and Welford, A.T. (ed.) London, 1954, pp. 5-16

43. Lippisch, A., "Das Luftskiff, Betrachtungen über den Menschenflug", Flugsport, 25, 1933, pp. 311-315

44. Lupton, H. "An Analysis of the Effects of Speed on the Mechanical Efficiency of Human Muscular Movement", Journ. of Physiol., 57, 1923, pp. 337-349

45. Lupton, H., "The Relation Between the External Work Produced and the Time Occupied in a Singular Muscular Contraction in Man", Journ. of Physiol., 57, 1923, pp. 68-75

46. Müller, E.A., "Der Einfluss von Pausen auf das Arbeitsmaximum beim Radfahren", Arbeitsphysiologie, Heft 3, 11, 1940, pp. 211-218

47. Müller, E.A., and Müller, A., "Die günstigte Grösse und Anordnung von Handradern", Arbeitsphysiologie, 14 (1), 1949, pp. 27-44

48. Olson, H.F., Dynamical Analogies, New York, D. Van Nostrand and Co., 1943, p. 196

49. Passmore, R., and Durnin, J.V.G.A., "Human Energy Expenditure", Physiological Reviews, 35, No. 4, 1955, pp. 801-840.

50. Rankine, W.J.M., A Manual of Applied Mechanics, 4th ed., London, 1868, p. 626

51. Raspet, A.W., "Human Muscle Powered Flight", Soaring, 16, 1952, pp. 18-19

52. Reach, Biochem. Ztschr., 14, 1908, p. 430 as quoted by Lupton(44)

53. Reed, J.D., "Factors Influencing Rotary Performance", Journ of Psychol., 28, 1949, pp. 65-92

54. Ross, D.M., The Use of Mechanical Principles in the Evaluation of a Human Functional Dimension, Ph.d Thesis, University of Pittsburgh, 1

55. Scotsman, October 2, 1901, as reported by Unna, P.J.H., "Limits of Effective Human Power", Nature, 158, 1946, pp. 560-561

56. Silva, C., Gli studi sul volo muscolare in Italia, Mitteilungsblatt No. 7 der Internationalan Studien Kommision für den Motorlosen Flug, 1938, p. 58

57. Ursinus, O., "Wenn Man ein Muskelflieg baut", Flugsport, 25, 1933, pp. 308-311

58. Ursinus, O., Mitteilungen des Muskelflug Institut, Bericht No. 1, 1935, 1935, p. 40 (as transmitted in personal communication to Ezra S. Krendel by A.W. Raspet)

59. Wilkie, D.R., "The Relation Between Force and Velocity in Human Muscle", Journ. of Physiol., 110, 1949, pp. 249-269

60. Wilkie, D.R., "The Circuit Analogue of Muscle", Electronic Engineering, October, 1950, pp. 435-437

61. Wilkie, D.R., "Facts and Theories about Muscle", Progress in Biophysics, 4, London, 1954, pp. 288-324

62. Zuntz and Shumburg, Physiologie des Marches, 1901, Berlin, p. 260 [as quoted in Benedict and Cathcart(3)]